AVANTHI INSTITUTE OF ENGINEERING AND TECHNOLOGY

Approved By AICTE, Permanently Affiliated to JNT University, Kakinada,Tamaram, Makavarapalem, Narsipatnam(R D), Visakhapatnam Dist-531113

Additional Information / Evidences sharing the procedure adopted for effective curriculum delivery in the Institute

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Website: www.jntuk.edu.in
Email: dap@jntuk.edu.in

Phone: 0884-2300991

Directorate of Academic Planning

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY KAKINADA
KAKINADA-533003, Andhra Pradesh, INDIA
(Established by AP Government Act No. 30 of 2008)

Lr. No. DAP/RAC/ II,III & IV Year /B. Tech/B. Pharmacy/2021 *Date 08.10.2021*

Dr. R. Srinivasa Rao,
Director, Academic Planning
JNTUK, Kakinada

To
All the Principals of Affiliated Colleges,
JNTUK. Kakinada.

Revised Academic Calendar for II, III, IV Year - B. Tech/B. Pharmacy for the AY 2021-22
(As per G.O. Rt. No. 242, Higher Education (U.E) Dept., dated 13.09.2021)

I SEMESTER			
Description	**From**	**To**	**Weeks**
Commencement of Class Work	**01.10.2021**		
I Unit of Instruction	01.10.2021	20.11.2021	7W
I Mid Examinations	22.11.2021	27.11.2021	1W
II Unit of Instructions	29.11.2021	15.01.2022	7W
II Mid Examinations	17.01.2022	22.01.2022	1W
Preparation & Practicals	24.01.2022	29.01.2022	1W
End Examinations	31.01.2022	12.02.2022	2W
Commencement of II Semester Class Work	**14.02.2022**		
II SEMESTER			
I Unit of Instructions	**14.02.2022**	02.04.2022	7W
I Mid Examinations	04.04.2022	09.04.2022	1W
II Unit of Instructions	11.04.2022	28.05.2022	7W
II Mid Examinations	30.05.2022	04.06.2022	1W
Preparation & Practicals	06.06.2022	11.06.2022	1W
End Examinations	13.06.2022	25.06.2022	2W
Commencement of next Year Class Work			
Note: Calendar is prepared with 8 hrs/day hence 7 weeks per instruction period			

R. Srinivasa Rao
Director Academic Planning

Director
Academic Planning
JNTUK Kakinada

Copy to the Secretary to the Hon'ble Vice Chancellor, JNTUK
Copy to Rector, Registrar, JNTUK
Copy to Director Academic Audit, JNTUK
Copy to Director of Evaluation, JNTUK

Principal
Avanthi Institute of Engg. & Technology
Tamaram, Makavarapalem Md.,
Visakhapatnam District., Pin-531113

AVANTHI INSTITUTE OF ENGINEERING & TECHNOLOGY

TAMARAM(V), MAKAVARAPALEM (M)
VISAKHAPATNAM-531113

ACADEMIC CALENDAR

Commencement of Class Work for IIB. Tech- I Sem: 01.10.2021

Description	From	To	Weeks
I Unit of Instructions	01.10.2021	20.11.2022	7W
I Mid Examinations	22.11.2021	27.11.2021	1W
II Unit of Instructions	29.11.2021	15.01.2022	7W
II Mid Examinations	17.01.2022	22.01.2022	1W
Preparation &Practicals	24.01.2022	29.01.2022	1W
End Examinations	31.01.2022	12.02.2022	2W

*Mid Examinations are to be conducted without effecting the regular class work

Week No	Date		No. of Working Days	Reports to be Submitted	Target Date
	From	To			
1	1-Oct-21	2-Oct-21	1	Monthly Attendance and Syllabus Completion report up to 30.10.2021 and student Counseling	On or before 01-Nov-2021
2	4-Oct-2021	9-Oct-21	6		
3	11-Oct-21	16-Oct-21	2		
4	18-Oct-2021	23-Oct-21	5		
5	25-Oct-21	30-Oct-21	6		
6	1-Nov-2021	6-Nov-21	5	Monthly Attendance and Syllabus Completion report up to 20.11.2021 and student Counseling	On or before 22-Nov-2021
7	8-Nov-21	13-Nov-21	6		
8	15-Nov-2021	20-Nov-21	6		
9	22-Nov-21	27-Nov-21	I- Mid & Online Examinations	Submission of Absentee Statement and Result Analysis	On or before 29-Nov-21
10	29-Nov-2021	4-Dec-21	6	Monthly Attendance and Syllabus Completion report up to 25.12.2021 and student Counseling	On or before 27-Dec-2021
11	6-Dec-21	11-Dec-21	6		
12	13-Dec-2021	18-Dec-21	6		
13	20-Dec-21	25-Dec-21	5		
14	27-Dec-2021	1-Jan-22	5	Monthly Attendance and Syllabus Completion report up to 12.01.2022 and student Counseling	On or before 17-Jan-2022
15	3-Jan-22	8-Jan-22	6		
16	10-Jan-2022	15-Jan-22	2		
17	17-Jan-22	22-Jan-22	II- Mid & Online Examinations	Submission of Absentee Statement and Result Analysis	On or before 24-Jan-2022
Submission of all Academic Documents Maintained by Faculty					

Total No. of. Working Days: 74
Expected Total No. of Periods per Subject: 80

Events to be organized

S. No	Name of the Event	Event Date
1	Induction Meet	First Week of Oct 2021
2	Industrial Visit	Last Week of Dec 2021

Principal

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AVANTHI INSTITUTE OF ENGINEERING & TECHNOLOGY

TAMARAM(V), MAKAVARAPALEM (M)
VISAKHAPATNAM-531113

ACADEMIC CALENDAR

Commencement of Class Work for IIB.Tech- II Sem: 14.02.2022

Description	From	To	Weeks
I Unit of Instructions	14.02.2022	02.04.2022	7W
I Mid Examinations	04.04.2022	09.04.2022	1W
II Unit of Instructions	11.04.2022	28.05.2022	7W
II Mid Examinations	30.05.2022	04.06.2022	1W
Preparation &Practicals	06.06.2022	11.06.2022	1W
End Examinations	13.06.2022	25.05.2022	2W

*Mid Examinations are to be conducted without effecting the regular class work

Week No	Date		No. of Working Days	Reports to be Submitted	Target Date
	From	To			
1	14-Feb-2022	19-Feb-22	6	Monthly Attendance and Syllabus Completion report up to 12.03.2022 and student Counseling	On or before 14-Mar-2022
2	21-Feb-22	26-Feb-22	6		
3	28-Feb-2022	5-Mar-22	6		
4	7-Mar-22	12-Mar-22	6		
5	14-Mar-2022	19-Mar-22	5	Monthly Attendance and Syllabus Completion report up to 02.04.2022 and student Counseling	On or before 04-Apr-2022
6	21-Mar-22	26-Mar-22	6		
7	28-Mar-2022	2-Apr-22	5		
8	4-Apr-22	9-Apr-22	I- Mid & Online Examinations	Submission of Absentee Statement and Result Analysis	On or before 11-Apr-22
9	11-Apr-2022	16-Apr-22	4	Monthly Attendance and Syllabus Completion report up to 07.05.2022 and student Counseling	On or before 09-May-2022
10	18-Apr-22	23-Apr-22	6		
11	25-Apr-2022	30-Apr-22	6		
12	2-May-22	7-May-22	6		
13	9-May-2022	14-May-22	6	Monthly Attendance and Syllabus Completion report up to 28.05.2022 and student Counseling	On or before 30-May-2022
14	16-May-22	21-May-22	6		
15	23-May-2022	28-May-22	6		
16	30-May-22	4-Jun-22	II- Mid & Online Examinations	Submission of Absentee Statement and Result Analysis	On or before 06-Jun-2022
Submission of all Academic Documents Maintained by Faculty					

Total No. of. Working Days: 80
Expected Total No. of Periods per Subject: 80

Events to be organized

S. No	Name of the Event	Event Date
1	Industrial Visit	3rd Week of Mar 2022
2	2-Day Technical Meet	3rd Week of April 2022
3	National symposium	3rd Week of May2022

Principal

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Visakhapatnam District., Pin: 531113

AVANTHI INSTITUTE OF ENGINEERING AND TECHNOLOGY

TAMARAM, MAKAVARAPALEM

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

II B.TECH - I SEMESTER(R20) ECE BRANCH TIME TABLE FOR THE ACADEMIC YEAR 2021-2022

W.E.F: 01 10 2021

DAY	1	2	3	12.30 - 02:00	4	5
	09:30-10:30	10:30-11:30	11:30-12:30		02:00-03:00	03:00-04:00
MON	SS	RVSP	EDC	BREAK	MEFA	STLD(T)
TUE	EDC	SS	RVSP		STLD	JAVA(T)
WED	MEFA	RVSP	EDC		STLD	JAVA
THU	SS	EDC	STLD		COUNSELLING/ REMIDIAL CLASSES	MEFA(T)
FRI	JAVA	RVSP	MEFA		SS	EDC(T)
SAT	STLD	DEPARTMENT ASSOC. MEETING			SS	LIBRARY/INTERNET

Electronic Devices & Circuits(EDC)	E Govinda	Object Oriented Programming Through Java(JAVA)	P M Manohar
Switching Theory & Logic Design(STLD)	P Sanyasi	Managerial Economics & Financial Analysis(MEFA)	P Ganesh
Signals & Systems(SS)	K Dhilli		
Random Variables & Stochastic Processes(RVSP)	R Prasad Rao		

HOD, ECE

HEAD OF THE DEPARTMENT
DEPARTMENT OF ECE
Avanthi Institute of Engg.&Tech.

Principal PRINCIPAL
Avanthi Institute of Engg. & Technology
Tamaram, Makavarapalem Md.,
Visakhapatnam District., Pin: 531113

AVANTHI INSTITUTE OF ENGINEERING AND TECHNOLOGY

TAMARAM (V), MAKAVAAPALEM (M), VISAKHAPATNAM – 531113

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

Program Name:	II B.TECH I SEM ECE 1&2	**AY**	2021-2022
Course Name:	RVSP	**Class / Sem**	II / I
Faculty Name:	Dr. R Prasad Rao		

Lecture No	Topic	Book / Web Reference	Teaching Method(s)
	THE RANDOM VARIABLE		
1.	"Review of probability theory, Definition of a Random Variable"	T3	C&T ; S/P
2.	Conditions for a Function to be a Random Variable	T3	C&T
3.	"Discrete, Continuous and Mixed Random Variables"	T2	C&T ; S/P
4.	Distribution and Density functions	T1	C&T ; CPS
5.	"Properties of Binomial, Poisson, Uniform, Gaussian"	TI	C&T
6.	Properties Exponential, Rayleigh	TI	C&T
7.	Properties Conditional Distribution, Conditional Density	TI	C&T ; GD
8.	REVISION		ASG ; Q
	OPERATION ON ONE RANDOM VARIABLE - EXPECTATIONS:		
9.	Introduction, Expected Value of a Random Variable	T1	C&T ; S/P
10.	Function of a Random Variable, Moments about the Origin	TI	C&T ; OL
11.	Central Moments, Variance and Skew	TI	C&T ; OL
12.	Chebychev's Inequality, Characteristic Function	TI	C&T
13.	Moment Generating Function, Transformations of a Random Variable	TI	C&T ; CPS
14.	Non-Monotonic Transformations for a Continuous Random Variable	TI	C&T
15.	REVISION		

	MULTIPLE RANDOM VARIABLES		
16.	Vector Random Variables, Joint Distribution Function Properties of Joint Distribution, Marginal Distribution Functions	**TI**	C&T ; OL
17.	Conditional Distribution and Density, Statistical Independence,Sum of Two Random Variables, Sum of Several Variables	**TI**	C&T ; SEM
	OPERATIONS ON MULTIPLE RANDOM VARIABLES:		
18.	Joint Moments about the Origin,Joint Central Moments, Joint Characteristic Functions	**TI**	C&T ; OL
19.	Jointly Gaussian Random Variables: Two Random Variables case, N Random Variables case	**T1**	C&T ; CPS
20.	Properties, Transformations of Multiple Random Variables	**TI**	C&T ; GD
21.	Linear Transformations of Gaussian Random Variables.	**TI**	C&T
	RANDOM PROCESSES – TEMPORAL CHARACTERISTICS:		
22.	The random process Concept, Classification of Processes	**T2**	C&T ; OL
23.	Deterministic and Nondeterministic Processes, Distribution and Density Functions	**T2**	C&T ; S/P
24.	Concept of Stationarity and Statistical Independence,	**T2**	C&T ; SEM
25.	First-Order Stationary Processes, Second-order and Wide-Sense Stationarity	**T2**	C&T ; OL
26.	Nth-order and Strict-Sense Stationarity, Time Averages and Ergodicity	**T2**	C&T
27.	Autocorrelation Function and its Properties, CrossCorrelation Function and its Properties,	**T2**	C&T ; GD
28.	Covariance Functions, Gaussian Random Processes, Poisson Random Process.	**T1**	C&T
	RANDOM PROCESSES - SPECTRAL CHARACTERISTICS:		
29.	The Power Density Spectrum: Properties	**T2**	C&T

30.	Relationship between Power Density Spectrum and Autocorrelation Function	T2	C&T ; S/P
31.	The Cross-Power Density Spectrum, Properties	T2	C&T. ASG
32.	Relationship between Cross-Power Density Spectrum and Cross-Correlation Function.	T2	C&T ; S/P
	LINEAR SYSTEMS WITH RANDOM INPUTS:		
33.	Random Signal Response of Linear Systems	T2	C&T ; OL
34.	System Response – Convolution, Mean and Mean-squared Value of System Response	T2	C&T ; SEM
35.	Autocorrelation Function of Response, Cross-Correlation Functions of Input and Output	T2	C&T
36.	Spectral Characteristics of System Response: Power Density Spectrum of Response	T2	C&T ; CPS
37.	CrossPower Density Spectra of Input and Output	T2	C&T
38.	Band pass, Band-Limited and Narrowband Processes	T2	C&T
39.	Properties.	T2	C&T
	REVISION		ASG ; GD

Teaching Methods:

C&T:-Chalk & Talk; S/P:-Slides/PPT; Videos; SEM: Seminar; Demo; CHART; ET/GL: Expert Talk/Guest Lecture; QUIZ; CPS: Class room problem solving; GD:-Group discussion; RTCS: Real time case studies; JAR:-Journal article review; PD:-Poster design; OL:-Online lecture/Google class room; Industrial Visit (IV), Assignment (ASG), Quiz/Puzzle (Q), Brain storming (BS), Think-Pair-Share (TPS), Certification(CERT), SIM: Simulation, P/G: Pledge/Greeting, Q/R: Quotes, references, LS: Literature Survey, RW: Report Writing, MM:Model making, PED: Professional/ethical dilemma, Coding, Activity/Event, FV: Filed Visit etc.

Text / Reference Books:

T-1: Probability, Random Variables & Random Signal Principles, Peyton Z. Peebles,TMH, 4th Edition,2001.

T-2: Probability, Random Variables and Stochastic Processes, Athanasios Papou lisand S.Unnikrisha, PHI, 4th Edition,2002.

T-3: Probability and Random Processes with Applications to Signal Processing, HenryStark and John W. Woods, Pearson Education, 3rd Edition,2001.

Web References:

W-1: NOC:Introduction to Probability Theory and Stochastic Processes, IIT Delhi
https://nptel.ac.in/courses/111102111

W-2: NOC:Probability and Random Variables/ Processes for Wireless Communications, IIT Kanpur
https://nptel.ac.in/courses/117104117

Signature of Course Coordinator

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY: KAKINADA
KAKINADA – 533 003, Andhra Pradesh, India

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

2021-22

II B.Tech. I Sem.

RANDOM VARIABLES & STOCHASTIC PROCESSES

UNIT I:

THE RANDOM VARIABLE : Introduction, Review of Probability Theory, Definition of a Random Variable, Conditions for a Function to be a Random Variable, Discrete, Continuous and Mixed Random Variables, Distribution and Density functions, Properties, Binomial, Poisson, Uniform, Gaussian, Exponential, Rayleigh, Conditional Distribution, Conditional Density, Properties.

UNIT II:

OPERATION ON ONE RANDOM VARIABLE – EXPECTATIONS : Introduction, Expected Value of a Random Variable, Function of a Random Variable, Moments about the Origin, Central Moments, Variance and Skew, Chebychev's Inequality, Characteristic Function, Moment Generating Function, Transformations of a Random Variable: Monotonic Transformations for a Continuous Random Variable, Non-monotonic Transformations of Continuous Random Variable.

UNIT III:

MULTIPLE RANDOM VARIABLES : Vector Random Variables, Joint Distribution Function, Properties of Joint Distribution, Marginal Distribution Functions, Conditional Distribution and Density, Statistical Independence, Sum of Two Random Variables, Sum of Several Random Variables, Central Limit Theorem: Unequal Distribution, Equal Distributions. OPERATIONS ON MULTIPLE RANDOM VARIABLES: Joint Moments about the Origin, Joint Central Moments, Joint Characteristic Functions, Jointly Gaussian Random Variables: Two Random Variables case, N Random Variables case, Properties, Transformations of Multiple Random Variables, Linear Transformations of Gaussian Random Variables.

UNIT IV:

RANDOM PROCESSES – TEMPORAL CHARACTERISTICS: The Random Process Concept, Classification of Processes, Deterministic and Nondeterministic Processes, Distribution and Density Functions, Concept of Stationarity and Statistical Independence. First-Order Stationary Processes, Second Order and Wide-Sense Stationarity, Nth -order and Strict-Sense Stationarity, Time Averages and Ergodicity, Autocorrelation Function and its Properties, Cross-Correlation Function and its Properties, Covariance Functions, Gaussian Random Processes, Poisson Random Process.

UNIT V:

RANDOM PROCESSES – SPECTRAL CHARACTERISTICS: The Power Spectrum: Properties, Relationship between Power Spectrum and Autocorrelation Function, The Cross-Power Density Spectrum, Properties, Relationship between Cross-Power Spectrum and Cross-Correlation Function.

LINEAR SYSTEMS WITH RANDOM INPUTS : Random Signal Response of Linear Systems: System Response – Convolution, Mean and Mean-squared Value of System Response, Autocorrelation Function of Response, Cross-Correlation Functions of Input and Output, Spectral Characteristics of System Response: Power Density Spectrum of Response, Cross-Power Density Spectra of Input and Output, Band pass, BandLimited and Narrowband Processes, Properties, Modeling of Noise Sources: Resistive (Thermal) Noise Source, Arbitrary Noise Sources, Effective Noise Temperature, Average Noise Figure, Average Noise Figure of cascaded networks

TEXTBOOKS:

1. Probability, Random Variables & Random SignalPrinciples, Peyton Z.Peebles, TMH, 4 thEdition, 2001.

2. Probability, Random Variables and Stochastic Processes, Athanasios Papoulis and S.Unnikrisha, PHI,4th Edition, 2002.

3. Probability and Random Processes with Applications to Signal Processing, Henry Starkand John W.Woods, Pearson Education, 3 rdEdition, 2001.

REFERENCE BOOKS:

1. Schaum's Outline of Probability, Random Variables, and Random Processes, 1997.

2. An Introduction to Random Signals and Communication Theory, B.P.Lathi, International Textbook, 1968.

3. Probability Theory and Random Processes, P. Ramesh Babu, McGrawHill,2015.

Course File Contents

Sr. No.	Contents
1.	Department V&M, PEO's, PO's
2.	Course Information Sheet
3.	University Calendar
4.	College Academic Calendar
5.	Time table
6.	Lesson plan
7.	Course Objectives
8.	Course Outcomes
9.	CO-PO mapping
10.	Curricular Gaps
11.	Topics beyond syllabus
12.	Lecture notes
13.	PPT's, Videos (in CD), Self-Learning Material
14.	Assignments
15.	Tutorial Handouts
16.	Unit wise Question bank
17.	Mid 1- Question papers
18.	Mid 1 - Question paper – Key
19.	Mid 2- Question papers

20.	Mid 2 - Question paper – Key
21.	University Question papers
22.	Remedial Classes
23.	Result Analysis (After Completion of course)
24.	Attendance Register -Teacher Log updated with signature of faculty and HOD
25.	Sample Answer Sheets
26.	Sample Assignment Sheets
27.	Sample Tutorial Sheets

Signature of the faculty

INSTITUTE VISION

To develop highly skilled professionals with ethics and human values.

INSTITUTE MISSION

- To produce component highly motivated engineers and management.
- To encourage innovation and still ethical practices.
- To impact quality education with industrial exposure and professional training.
- To exhort the spirit of professional beyond academic excellence.

Department vision

We envision the department to make an impact on, and lead in the field of Electronics communications engineering through its education and research agenda

DEPARTMENT MISSION

"To produce highly competent electronics and communications engineers to suite global needs.

AVANTHI INSTITUTE OF ENGINEERING AND TECHNOLOGY

TAMARAM, MAKAVARAPALEM, NARSIPATNAM

SUBJECT: RVSP YEAR: II SEM: I BRANCH: ECE

PROGRAM EDUCATIONAL OBJECTIVES

- PEO 1: Graduates will have demonstrated a through grounding in the fundamental principles of basic sciences, English other engineering disciplines, Mechanical engineering subjects and practices convey a professional attitude appropriate for a diverse world community.
- PEO 2: Graduates will have undertaken complex problems and develop appropriate technical solutions.
- PEO 3: Graduates will be prepared to communicate and work effectively on team based engineering projects and will practice the ethics of their profession consistent with a series social responsibility.
- PEO 4: Graduates will have demonstrated exposure to emerging and modern technologies to succeed in engineering position in various agencies and also pursue advanced degrees in engineering.

PROGRAM OUTCOMES

1. **ENGINEERING KNOWLEDGE**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

2. **PROBLEM ANALYSIS**: Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

3. **DESIGN/DEVELOPMENT OF SOLUTIONS**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

4. **CONDUCT INVESTIGATIONS OF COMPLEX PROBLEMS**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

5. **MODERN TOOL USAGE**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modelling to complex engineering activities with an understanding of the limitations.

6. **THE ENGINEER AND SOCIETY**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

7. **ENVIRONMENT AND SUSTAINABILITY**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

8. **ETHICS**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

9. **INDIVIDUAL AND TEAM WORK**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

10. **COMMUNICATION**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, give and receive clear instructions.

11. **PROJECT MANAGEMENT AND FINANCE**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. **LIFE-LONG LEARNING**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

COURSE INFORMATION SHEET

PROGRAMME: B. TECH ECE Academic Year :2021-22	DEGREE: B.TECH II-I
COURSE: RANDOM VARIABLES & STOCHASTIC PROCESSES	SEMESTER: I CREDITS: 3
COURSE CODE: R2021044 REGULATION:R20	COURSE TYPE: CORE
COURSE AREA/DOMAIN: RANDOM VARIABLES & STOCHASTIC PROCESSES	CONTACT HOURS: 3+1 (Tutorial) hours/Week.
CORRESPONDING LAB COURSE CODE (IF ANY): No	LAB COURSE NAME: No

SYLLABUS:

UNIT	DETAILS	HOURS
I	**THE RANDOM VARIABLE:** Introduction, Review of Probability Theory, Definition of a Random Variable, Conditions for a Function to be a Random Variable, Discrete, Continuous and Mixed Random Variables, Distribution and Density functions, Properties, Binomial, Poisson, Uniform, Gaussian, Exponential, Rayleigh, Conditional Distribution, Conditional Density, Properties.	12
II	**OPERATION ON ONE RANDOM VARIABLE – EXPECTATIONS:** Introduction, Expected Value of a Random Variable, Function of a Random Variable, Moments about the Origin, Central Moments, Variance and Skew, Chebychev's Inequality, Characteristic Function, Moment Generating Function, Transformations of a Random Variable: Monotonic Transformations for a Continuous Random Variable, Non-monotonic Transformations of Continuous Random Variable.	12
III	**MULTIPLE RANDOM VARIABLES:** Vector Random Variables, Joint Distribution Function, Properties of Joint Distribution, Marginal Distribution Functions, Conditional Distribution and Density, Statistical Independence, Sum of Two Random Variables, Sum of Several Random Variables, Central Limit Theorem: Unequal Distribution, Equal Distributions. **OPERATIONS ON MULTIPLE RANDOM VARIABLES:** Joint Moments about the Origin, Joint Central Moments, Joint Characteristic Functions, Jointly Gaussian Random Variables: Two Random Variables case, N Random Variables case,	11

	Properties, Transformations of Multiple Random Variables, Linear Transformations of Gaussian Random Variables.	
IV	**RANDOM PROCESSES – TEMPORAL CHARACTERISTICS:** The Random Process Concept, Classification of Processes, Deterministic and Nondeterministic Processes, Distribution and Density Functions, Concept of Stationarity and Statistical Independence. First-Order Stationary Processes, Second- Order and Wide-Sense Stationarity, Nth-order and Strict-Sense Stationarity, Time Averages and Ergodicity, Autocorrelation Function and its Properties, Cross-Correlation Function and its Properties, Covariance Functions, Gaussian Random Processes, Poisson Random Process.	11
V	**RANDOM PROCESSES – SPECTRAL CHARACTERISTICS:** The Power Spectrum: Properties, Relationship between Power Spectrum and Autocorrelation Function, The Cross-Power Density Spectrum, Properties, Relationship between Cross-Power Spectrum and Cross-Correlation Function. **LINEAR SYSTEMS WITH RANDOM INPUTS:** Random Signal Response of Linear Systems: System Response – Convolution, Mean and Mean-squared Value of System Response, Autocorrelation Function of Response, Cross-Correlation Functions of Input and Output, Spectral Characteristics of System Response: Power Density Spectrum of Response, Cross-Power Density Spectra of Input and Output, Band pass, Band- Limited and Narrowband Processes, Properties	11 13
	TOTAL HOURS	70

TEXT/REFERENCE BOOKS:

T/R	BOOK TITLE/AUTHORS/PUBLICATION
1	Probability, Random Variables & Random Signal Principles, Peyton Z. Peebles, TMH, 4th Edition, 2001.
2	Probability, Random Variables and Stochastic Processes, Athanasios Papoulis and S.Unnikrisha, PHI, 4th Edition, 2002.
3	Probability Theory and Stochastic Processes – B. Prabhakara Rao, Oxford University Press
4	Probability and Random Processes with Applications to Signal Processing, Henry Stark and John W.Woods, Pearson Education, 3rd Edition.
5	Probabilistic Methods of Signal & System Analysis, George R. Cooper, Clave D. Mc Gillem, Oxford, 3rd Edition, 1999.
6	Statistical Theory of Communication, S.P.Eugene Xavier, New Age Publications, 2003
7	Signals, Systems & Communications, B.P. Lathi, B.S. Publications, 2003

8	Probability and Random Processes, An Introduction for Applied Scientists and Engineers, Davenport W.B, McGraw-Hill, 1970.
9	Introduction to Random Processes with Applications to Signals and Systems, Gardener W.A, McGraw-Hill, 2nd Edition
10	Schaum's Outline of Probability, Random Variables, and Random Processes.
11	An Introduction to Random Signals and Communication Theory, B.P. Lathi, International Textbook, 1968.

COURSE PRE-REQUISITES:

C.CODE	COURSE NAME	DESCRIPTION	SEM
	Mathematics I&II	Basic concepts about mathematics	I-I
	P&S	Probability and statistics	

COURSE OBJECTIVES:

1	To give students an introduction to elementary probability theory, in preparation to learn the concepts of statistical analysis, random variables and stochastic processes
2	To mathematically model theory and phenomena with the help of probability theory Concepts
3	To introduce the important concepts of random variables and stochastic processes.
4	To analyze the LTI systems with stationary random process as input.

COURSE OUTCOMES:

SNO	DESCRIPTION
1	Identify random variables and Define distribution and density functions. (L1)
2	Determine the properties of a random variable from its probability density and distribution functions. (L3)
3	Illustrate the changes in the properties of random variables upon combining them with other random variables. (L4)
4	Differentiate stochastic and ergodic processes. (L2)
5	Measure the covariance and spectral density of stationary random processes. (L5)

GAPS IN THE SYLLABUS - TO MEET INDUSTRY/PROFESSION REQUIREMENTS:

SNO	DESCRIPTION	PROPOSED ACTIONS
1	Probability theory	GUEST LECTURE
2	Rectangular destructive functions	NPTEL LECTURES
3	Applications of RVSP in signal processing and communication systems	NPTEL LECTURES
4	Some other topics in noise like addition of noise due to several amplifiers, equivalent noise temperature of cascaded stages.	GUEST LECTURER

PROPOSED ACTIONS: TOPICS BEYOND SYLLABUS/ASSIGNMENT/INDUSTRY VISIT/GUEST LECTURER/NPTEL ETC

TOPICS BEYOND SYLLABUS/ADVANCED TOPICS/DESIGN:

1	Applications of PTSP in signal processing and communication systems
2	Rectangular destructive functions

WEB SOURCE REFERENCES:

1	http://www.math.uiuc.edu/
2	http:// *nptel.iitm.ac.in/...**random**.../NPT39*

DELIVERY/INSTRUCTIONAL METHODOLOGIES:

☐ CHALK & TALK	☐ STUD. ASSIGNMENT	☐ WEB RESOURCES	
LCD/SMART BOARDS	☐ STUD. SEMINARS	☐ ADD-ON COURSES	

ASSESSMENT METHODOLOGIES-DIRECT

ASSIGNMENTS	☐ STUD.SEMINARS	☐ TESTS/MODEL EXAMS	☐ UNIV. EXAMINATION
☐ STUD. LAB PRACTICES	☐ STUD. VIVA	☐ MINI/MAJOR PROJECTS	☐ CERTIFICATIONS
☐ ADD-ON COURSES			☐ OTHERS

ASSESSMENT METHODOLOGIES-INDIRECT

☐ ASSESSMENT OF COURSE OUTCOMES (BY FEEDBACK, ONCE)	☐ STUDENT FEEDBACK ON FACULTY (TWICE)
☐ ASSESSMENT OF MINI/MAJOR PROJECTS BY EXT. EXPERTS	☐ OTHERS

AVANTHI INSTITUTE OF ENGINEERING & TECHNOLOGY

TAMARAM, MAKAVARAPALEM, NARSIPATNAM

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

Subject: RANDAM VARIABLES & STOCHASTIC PROCESSES

Branch: ECE II year I sem.

COURSE OBJECTIVE

1	To give students an introduction to elementary probability theory, in preparation to learn the concepts of statistical analysis, random variables and stochastic processes
2	To mathematically model theory and phenomena with the help of probability theory Concepts
3	To introduce the important concepts of random variables and stochastic processes.
4	To analyze the LTI systems with stationary random process as input.

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COURSE OUTCOMES

SNO	DESCRIPTION
CO 1	Identify random variables and Define distribution and density functions. (L1)
CO 2	Determine the properties of a random variable from its probability density and distribution functions. (L3)
CO 3	Illustrate the changes in the properties of random variables upon combining them with other random variables. (L4)
CO 4	Differentiate stochastic and ergodic processes. (L2)
CO 5	Measure the covariance and spectral density of stationary random processes. (L5)
CO 6	Formulate the response of the LTI system with random excitation based on the concepts of Signals and systems. (L6)

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CO-PO Mapping

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	2	2	-	-	-	-	-	-	-	-	-	-
CO 2		3	-	-	-	-	-	-	-	-	-	-
CO 3				2	-	-	-	-	-	-	-	-
CO 4			2	-	-	-	-	-	-	-	-	-
CO 5		2	-	-	-	-	-	-	-	-	-	-
CO 6				3	1	-	-	-	-	-	-	-

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

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Branch: ECE II year I sem.

PEO'S AND PO'S MAPPING

SNO	DESCRIPTION	PEO'S MAPPING	PO'S MAPPING
1	Simple probabilities using an appropriate sample space.	PEO1,PEO2,PEO3,PEO4	1,3,4
2	Simple probabilities and expectations from probability density functions (pdfs)	PEO1,PEO2,PEO3,PEO4	1,3,4
3	Likelihood ratio tests from pdfs for statistical engineering problems.	PEO1,PEO2,PEO3,PEO4	1,3,4,5
4	Least – square & maximum likelihood estimators for engineering problems.	PEO1,PEO2,PEO3,PEO4	1,3,4,5
5	Mean and covariance functions for simple random processes.	PEO1,PEO2,PEO3,PEO4	1,3,4

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Branch: ECE II year I sem.

CURRICULAR GAPS

Process:

The concerned faculty will verify the syllabus and suggest the missing contents and they will approach the senor faculties of the department to go through the syllabus prescribed the university in detail

Curricular gaps:

SNO	DESCRIPTION
1	Probability theory
2	Rectangular destructive functions
3	Applications of RVSP in signal processing and communication systems
4	Some other topics in noise like addition of noise due to several amplifiers, equivalent noise temperature of cascaded stages.

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

Subject: RANDAM VARIABLES & STOCHASTIC PROCESSES

Branch: ECE II year I sem.

TOPICS BEYOND THE SYLLABUS

1	Applications of PTSP in signal processing and communication systems
2	Rectangular destructive functions

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ASSESSMENT OF LEARNING OUTCOMES

Random Variables and Stochastic Processes

CAY : 2021-22	SEM : ☑ODD☐EVEN	Date : 22/1/2022
SUBJECT	**Random Variables and Stochastic Processes**	Year : ☐I☑II☐III ☐IV
FACULTY	R.PRASAD RAO	

CAY: CURRENT ACADEMIC YEAR

SPECIFIED LEARNING OUTCOMES FOR SUBJECT – Random Variables and Stochastic Processes

1. Identify random variables and Define distribution and density functions.
2. Determine the properties of a random variable from its probability density and distribution functions.
3. Illustrate the changes in the properties of random variables upon combining them with other random variables.
4. Differentiate stochastic and ergodic processes.
5. Measure the covariance and spectral density of stationary random processes.
6. Formulate the response of the LTI system with random excitation based on the concepts of Signals and systems.

Please evaluate on the following Scale:

Excellent(E)	Good(G)	Average(A)	Poor(P)	No Comment(NC)
5	4	3	2	1

SNO	QUESTIONNAIRE	E 5	G 4	A 3	P 2	NC 1
GENERAL OBJECTIVES:						
1)	Was this course able to give you an introduction to elementary probability theory, in preparation for learning the concepts of statistical analysis, random variables, and stochastic processes	☐	☑	☐	☐	☐
2)	Can you mathematically model theory and phenomena with the help of probability theory Concepts?	☑	☐	☐	☐	☐

SPECIFIC **LEARNING OUTCOMES:**		
3)	Can you identify random variables and Define distribution and density functions?	☐ ☑ ☐ ☐ ☐
4)	Can you determine the properties of a random variable from its probability density and distribution functions?	☐ ☑ ☐ ☐ ☐
5)	Can you Illustrate the changes in the properties of random variables upon combining them with other random variables?	☐ ☑ ☐ ☐ ☐
6)	Can you differentiate stochastic and ergodic processes?	☐ ☑ ☐ ☐ ☐
7)	Can you measure the covariance and spectral density of stationary random processes?	☑ ☐ ☐ ☐ ☐
8)	Can you formulate the response of the LTI system with random excitation based on the concepts of Signals and systems?	☐ ☑ ☐ ☐ ☐

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1. The Random Variable (Assignment - 1)

1(a) Define and Explain the following with an example :
(i) Discrete Sample Space (ii) Conditional Probability
(iii) Continuous Random Variable (iv) Conditional Density Function.

Ans (i) Discrete Sample Space :-

A Sample Space may contain a Number of Outcomes that depends on the experiment. If it Contains a finite Number of Outcomes then it is Known as Discrete (or) Finite Sample Space.

Eg:- In tossing a Coin
ie $S = \{H, T\}$
In Rolling a dice
$S = \{1, 2, 3, 4, 5, 6\}$

(ii) Conditional Probability :-

For two Events A and B then the Conditional Probability of Event A, assuming that Event B has happened

ie $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ provided $p(B) \neq 0$

Similarly the conditional Probability of Event B, assuming that Event A has happened

ie $P(B/A) = \frac{P(A \cap B)}{P(B)}$ provided $p(A) \neq 0$

Eg:- To the Fair die Experiment the conditional probability of getting 1. Given that odd Number faces is Obtained.

Sample Space $S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

Event $A = \{1\}$, $n(A) = 1$

Event $B = \{1, 3, 5\}$, $n(B) = 3$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$A \cap B = \{1\}, \; n(A \cap B) = 1$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{6}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{6}$$

$$P(A|B) = \frac{1/6}{3/6} = \frac{1}{3}$$

(iii) Continuous Random Variable :-

If a Random Variable takes Infinite Number of uncountable values is Known as Continuous Random Variable

Eg:- The Length of Time During which Vaccum tube is Installed in the Circuit Function i.e 0-120 sec

(iv) Conditional Density Function :-

The conditional density Function of the Random Variable x is defined as the derivative of the conditional distribution Function.

$$f_x(x) = \frac{d}{dx} F_x(x)$$

1(b) Two Boxes are selected Randomly. The 1st Box contains 2 White Balls & 3 black Balls, The 2nd Box contains 3 white balls and 4 Black Balls. What is the probability of drawing a White Ball.

Sol

Let

Box 1 is represented by A

Box 2 is represented by B

The Probability of Selecting any Ball is

$$P(A) = P(B) = \frac{1}{2}$$

Box A Contains 2 White Balls & 3 Black Balls. then,

Probability of Selecting White balls given that Box A is choosen ie $P(W/A)$

$$\Rightarrow P(W/A) = \frac{2}{5}$$

Box B Contains 3 white balls and 4 Black Balls then,

Probability of Selecting White balls given that Box B is choosen ie $P(W/B)$

$$\Rightarrow P(W/B) = \frac{3}{7}$$

→ By Using Total Probability Theorem,

Probability of drawing a White ball is $P(W)$

then, $P(W) = P(A)P(W/A) + P(B)P(W/B)$

$$= \left(\frac{1}{2}\right)\left(\frac{2}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{7}\right)$$

$$= \frac{14+15}{70}$$

$$= \frac{29}{70}$$

$$\therefore P(W) = \frac{29}{70}$$

Q(a) State & Prove the Properties of Cummulative Distribution Function (CDF) & Probability Density Function (PDF) of x.

Ans Properties of CDF :-

1st property :- $F_X(x)$ is non decreasing Function of Random Variable 'x'

ie If $x_1 < x_2$ then $F_X(x_1) < F_X(x_2)$

2nd property :- $F_X(-\infty) = 0$

As per the definition $F_X(-\infty) = P(x \leq -\infty) = 0$, x takes values in the Range of "$-\infty$ to $+\infty$". There are no Real Number $(x \leq -\infty)$

3rd property :- $F_X(-\infty) = 1$

As per the definition

$$F_X(\infty) = P(x \leq \infty)$$
$$= P(-\infty < x \leq \infty)$$
$$= P(S)$$
$$= 1$$

4th Property :- $0 \leq F_X(x) \leq 1$

$\therefore F_X(x)$ is also an Probability Function.

5th Property :- If x is discrete Random Variable

$$x_1 < x_2 ---- < x_{i-1} < x_i < ---$$

$$P(x = x_i) = F_X(x_i) - F_X(x_{i-1})$$
$$= P(x \leq x_i) - P(x \leq x_{i-1})$$

6th Property :- If x be the Random Variable then

$$P(x_1 < x < x_2) = F_X(x_2) - F_X(x_1)$$

$$P(x_1 < x \leq x_2) = \int_{x_1}^{x_2} f_X(x)\,dx$$
$$= F_X(x)\Big|_{x_1}^{x_2}$$
$$= F_X(x_2) - F_X(x_1)$$

7th Property :- If X be the Random Variable then,

$$P(X>x) = 1 - F_X(x)$$
$$= 1 - P(X \le x)$$

Proof :- $(X>x) \cup (X \le x) = S$

Taking Probabilities on Both Sides

$$\Rightarrow P\left[(X>x) \cup (X \le x)\right] = P(S)$$

$$\Rightarrow P(X>x) + P(X \le x) = 1$$

$$\Rightarrow P(X>x) = 1 - P(X \le x)$$
$$= 1 - F_X(x).$$

Properties of PDF :-

1st property :- $f_X(x) \ge 0$ for all the values of x

$\therefore f_X(x)$ is also a probability Function.

2nd property :- $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

Proof :- $\int_{-\infty}^{\infty} f_X(x)\,dx = F_X(x)\Big|_{-\infty}^{\infty}$

$$= F_X(\infty) - F_X(-\infty)$$
$$= 1 - 0$$
$$= 1$$

$\therefore \int_{-\infty}^{\infty} f_X(x)\,dx = 1$

3rd property :- If x is Continous Random Variable then

$$f_X(x) = \frac{d}{dx} F_X(x)$$

4th Property :- If x is Discrete Random Variable then

$$f_X(x) = \sum_{n=1}^{N} P(x_i)\,\delta(x - x_i)$$

Proof :- $f_X(x) = \frac{d}{dx} F_X(x) - \frac{d}{dx}\left[\sum_{i=1}^{N} P(x_i)\,u(x - x_i)\right]$

$$= \sum_{i=1}^{N} P(x_i) \frac{d}{dx} u(x - x_i)$$

$$= \sum_{i=1}^{N} p(x_i)\,\delta(x-x_i)$$

26. The Random Variable 'x' has the discrete Variable in the set $\{-1, -0.5, 0.7, 1.5, 3\}$ the corresponding probabilities are assumed to be $(0.1, 0.2, 0.1, 0.4, 0.2\}$. Plot its CDF and state it is a discrete (or) Continous Distribution Function.

Sol Probability distribution Function Table;

x	-1	-0.5	0.7	1.5	3
$P(x)$	0.1	0.2	0.1	0.4	0.2

If x is discrete Random Variable then

$$F_X(x) = P(X \le x)$$

If $x = -1$, $F_X(-1) = P(X \le -1) = P(X = -1) = 0.1$

If $x = -0.5$, $F_X(-0.5) = P(X \le -0.5)$

$$= P(X = -1) + P(X = -0.5)$$
$$= 0.1 + 0.2$$
$$= 0.3$$

If $x = 0.7$, $F_X(0.7) = P(X \le 0.7)$

$$= P(X = -1) + P(X = -0.5) + P(X = 0.7)$$
$$= 0.1 + 0.2 + 0.1$$
$$= 0.4$$

If $x = 1.5$, $F_X(1.5) = P(X \le 1.5)$

$$= P(x = -1) + P(x = -0.5) + P(x = 0.7) + P(x = 1.5)$$
$$= 0.1 + 0.2 + 0.1 + 0.4$$
$$= 0.8$$

If $x=3$, $F_x(3) = P(x \le 3)$

$$= p(x=-1) + p(x=-0.5) + p(x=0.7) + p(x=1.5) + p(x=3)$$

$$= 0.1 + 0.2 + 0.1 + 0.4 + 0.2$$

$$= 1.0$$

$$F_x(x) = \sum_{i=1}^{N} p(x_i)\, u(x-x_i)$$

$$= \sum_{i=1}^{5} p(x_i)\, u(x-x_i)$$

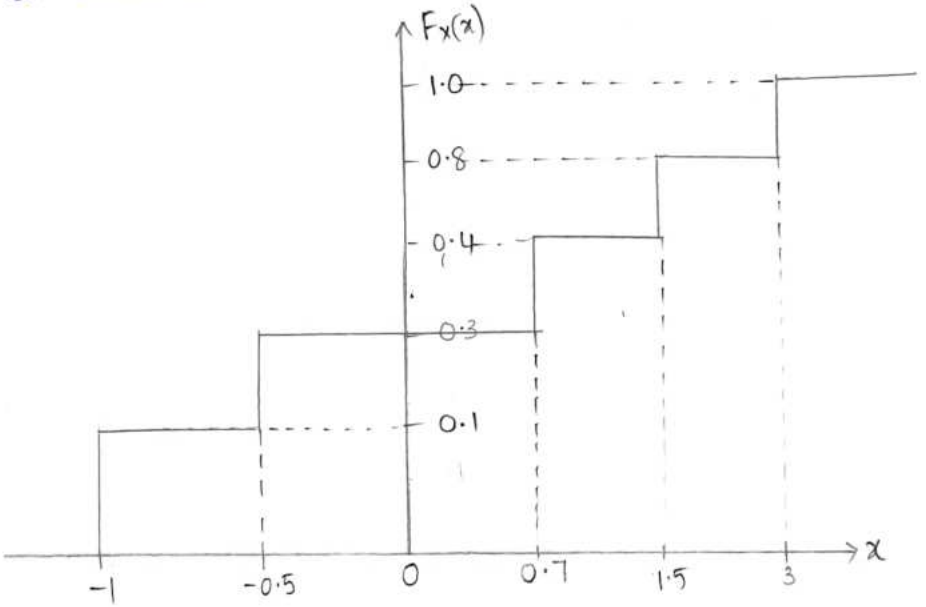
$$= p(x_1)u(x-x_1) + p(x_2)u(x-x_2) + p(x_3)u(x-x_3) + p(x_4)u(x-x_4) + p(x_5)u(x-x_5)$$

$$= p(-1)u(x+1) + p(-0.5)u(x+0.5) + p(0.7)u(x-0.7) + p(1.5)\,u(x-1.5) + p(3)u(x-3)$$

$$= 0.1\,u(x+1) + 0.2\,u(x+0.5) + 0.1\,u(x-0.7) + 0.4\,u(x-1.5) + 0.2\,u(x-3)$$

* It is a Discrete Distribution Function.

<u>Plot</u> :-



3(a) In an Experiment there are 2 Boxes. Each box Contains balls as Shown. The Event is to "Select a Box Randomly and then Select a Ball from the Selected Box". If the probability of Selecting 1st Box is 0.3 then find

(i) Conditional Probability Distribution & Density Functions

(ii) Probability Distribution & Density Functions

(iii) Plot the Functions

x_i	Ball Colour	BOXES 1	BOXES 2	
1	Red	20	40	60
2	Blue	30	30	60
3	Green	50	30	80
	Total	100	100	200

8d Let the 1st Box be B_1
& 2nd Box be B_2

Given Data
probability of Selecting first Box ie $p(B_1) = 0.3$
Using AXIOM 2

$$P(S) = 1$$

$$\Rightarrow p(B_1) + p(B_2) = 1$$

$$p(B_2) = 1 - p(B_1)$$

$$p(B_2) = 1 - 0.3$$

$$p(B_2) = 0.7$$

Now, define a discrete Random Variable 'x' to have values

$$x_1 = 1;\ x_2 = 2;\ x_3 = 3.$$

When Red, Blue, Green are Selected Respectively From table,
The conditional probabilities are.

* The Probability of getting a Red Ball When 'B_1' is Selected is

$$P\left(\frac{x_1}{B_1}\right) = \frac{20}{100} = 0.2$$

* The Probability of getting a Blue Ball When 'B_1' is Selected is

$$P\left(\frac{x_2}{B_1}\right) = \frac{30}{100} = 0.3$$

* The probability of getting a Green Ball When 'B_1' is Selected

$$P\left(\frac{x_3}{B_1}\right) = \frac{50}{100} = 0.5$$

Simillarly,

Conditional Density Function of 'B_2' is given by,

$$f_X(x/B_2) = \frac{d}{dx} F_X\left(\frac{x}{B_2}\right)$$

$$= \sum_{i=1}^{3} P(x_i/B_2)\,\delta(x-x_i)$$

$$= P\left(\frac{x_1}{B_2}\right)\delta(x-x_1) + P\left(\frac{x_2}{B_2}\right)\delta(x-x_2) + P\left(\frac{x_3}{B_2}\right)\delta(x-x_3)$$

$$= P\left(\frac{x_1}{B_2}\right)\delta(x-1) + P\left(\frac{x_2}{B_2}\right)\delta(x-2) + P\left(\frac{x_3}{B_3}\right)\delta(x-3).$$

$$= 0.4\,\delta(x-1) + 0.3\,\delta(x-2) + 0.4\,\delta(x-3)$$

(ii) Using Total Probability Theorem.

$$P(X_1) = P(B_1)P\left(\frac{X_1}{B_1}\right) + P(B_2)P\left(\frac{X_1}{B_2}\right)$$

$$= (0.3)(0.2) + (0.7)(0.4)$$

$$= 0.06 + 0.28$$

$$= 0.34$$

$$P(X_2) = P(B_1)P(X_2/B_1) + P(B_2)P(X_2/B_2)$$

$$= (0.3)(0.3) + (0.7)(0.3)$$

$$= 0.09 + 0.21$$

$$= 0.30$$

$$P(X_3) = P(B_1)P(X_3/B_1) + P(B_2)P(X_3/B_2)$$

$$= (0.3)(0.5) + (0.7)(0.4)$$

$$= 0.15 + 0.28$$

$$= 0.36$$

→ Distribution Function;

$$F_X(x) = \sum_{i=1}^{3} P(x_i)\,u(x-x_i)$$

$$= P(x_1)u(x-x_1) + P(x_2)u(x-x_2) + P(x_3)u(x-x_3)$$

Simillarly,

* The probability of getting a Red Ball when 'B_2' is Selected is

$$P\left(\frac{x_1}{B_2}\right) = \frac{40}{100} = 0.4$$

* The probability of getting a blue ball when 'B_2' is Selected

$$P\left(\frac{x_2}{B_2}\right) = \frac{30}{100} = 0.3$$

* The probability of getting a green ball when 'B_2' is Selected

$$P\left(\frac{x_3}{B_2}\right) = \frac{40}{100} = 0.4$$

(i) Conditional Distribution Function of 'B_1' is given by,

$$F_X\left(\frac{x}{B_1}\right) = \sum_{i=1}^{3} P\left(\frac{x_i}{B_i}\right) u(x-x_i)$$

$$= P\left(\frac{x_1}{B_1}\right) u(x-x_1) + P\left(\frac{x_2}{B_2}\right) u(x-x_2) + P\left(\frac{x_3}{B_3}\right) u(x-x_3)$$

$$= 0.2\, u(x-1) + 0.3\, u(x-2) + 0.5\, u(x-3)$$

Simillarly,

Conditional Distribution Function of 'B_2' is given by,

$$F_X\left(\frac{x}{B_2}\right) = \sum_{i=1}^{3} P\left(\frac{x_i}{B_2}\right) u(x-x_i)$$

$$= P\left(\frac{x_1}{B_2}\right) + u(x-x_1) + P\left(\frac{x_2}{B_2}\right) u(x-x_2) + P\left(\frac{x_3}{B_2}\right) u(x-x_3)$$

$$= 0.4\, u(x-1) + 0.3\, u(x-2) + 0.3\, u(x-3)$$

→ Conditional Density Function of 'B_1' is given by

$$f_X\left(\frac{x}{B_1}\right) = \frac{d}{dx} F_X\left(\frac{x}{B_1}\right)$$

$$= \sum_{i=1}^{3} P\left(\frac{x_i}{B_i}\right) \delta(x-x_i)$$

$$= P\left(\frac{x_1}{B_1}\right) \delta(x-x_1) + P\left(\frac{x_2}{B_1}\right) \delta(x-x_2) + P\left(\frac{x_3}{B_1}\right) \delta(x-x_3)$$

$$= 0.2\, \delta(x-1) + 0.3\, \delta(x-2) + 0.5\, \delta(x-3)$$

$$= 0.34\,u(x-1) + 0.30\,u(x-2) + 0.36\,u(x-3)$$

→ Density Function.

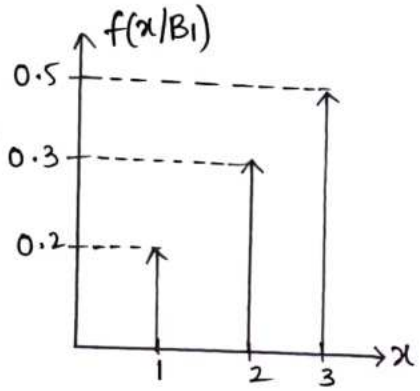
$$f_X(x) = \sum_{i=1}^{3} P(x_i)\,\delta(x-x_i)$$

$$= P(x_1)\,\delta(x-x_1) + P(x_2)\,\delta(x-x_2) + P(x_3)\,\delta(x-x_3)$$

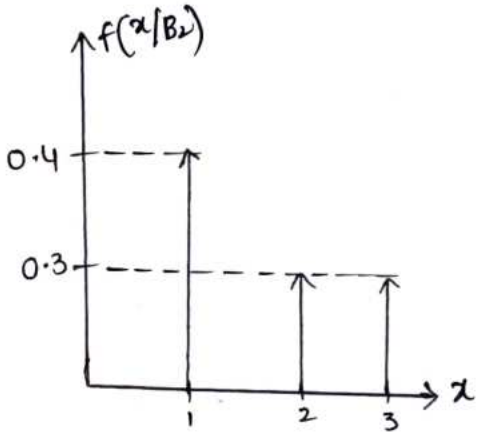
$$= 0.34\,\delta(x-1) + 0.30\,\delta(x-2) + 0.36\,\delta(x-3)$$

(iii) Plotting For Conditional Density Functions;

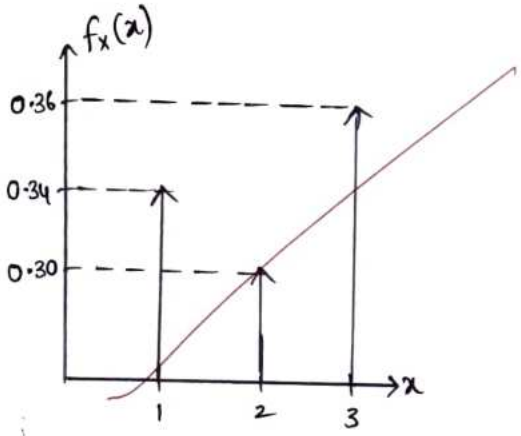
* $f(x/B_1)$



* $f(x/B_2)$

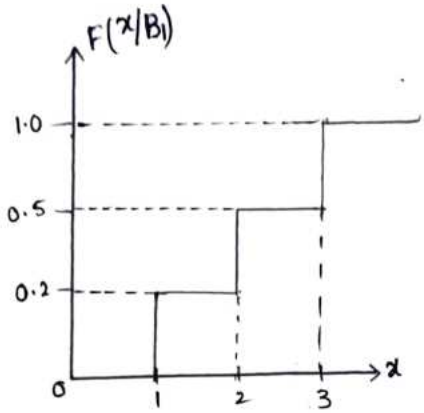


* $f_X(x)$

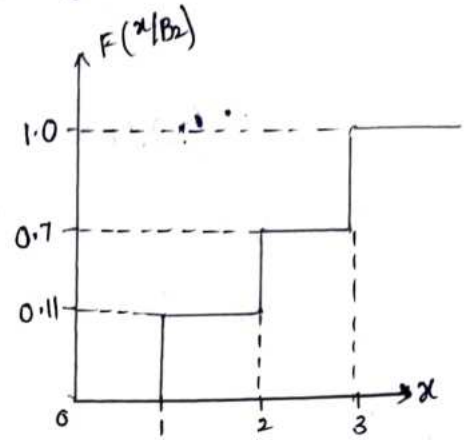


Plotting for conditional Distribution Functions.

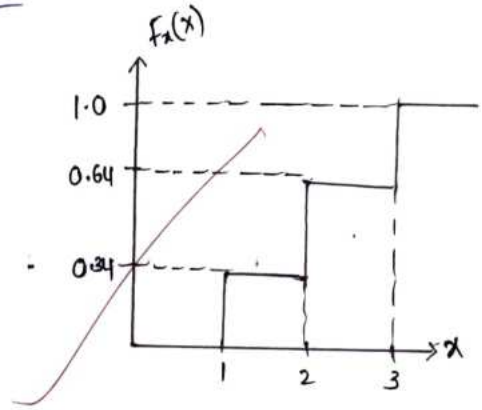
* $F(x/B_1)$:-



* $F(x/B_2)$:-



* $F_X(x)$:-



Assignment 2

(n) 21815A0429

UNIT - 2

1) a) State and Prove the Properties of variance of a random variable.

Ans:- Properties of variance:

1) The variance of a constant is zero, i.e if 'K' is a constant, then $var[K]=0$

Proof: It is known that $var[x] = E[(x-\bar{x})^2]$

$\boxed{\text{If } X=K}$ $\quad var[K] = E[(K-E(K))^2]$

$= E[(K-K)^2] = E[0] = 0$

2) If a is any arbitary constant then, $var[ax] = a^2\, var[x]$

Proof:- It is known that $var[x] = E[(x-\bar{x})^2]$

$var[x] = E[x^2] - [E[x]]^2$

$var[ax] = E[(ax)^2] - \{E[ax]\}^2$

$= a^2 E[x^2] - \{a\, E[x]\}^2$

$= a^2 [E[x^2] - \{E[x]\}^2]$

$= a^2\, var[x]$

$\therefore var[ax] = a^2\, var[x]$

3) If a and b are any two arbitary constants then,

$var[ax+b] = a^2\, var[x]$

Proof: $var[ax+b] = E[(ax+b)^2 - (E(ax+b))]^2$

$= E[a^2x^2 + b^2 + 2abx] - [a^2[E[x]]^2 + b^2 + 2ab\,E[x]]$

$= a^2 E[x^2] + \cancel{b^2} + \cancel{2ab\,E[x]} - a^2 [E(x)]^2 - \cancel{b^2} - \cancel{2ab\,E[x]}$

$= a^2 E[x^2] - a^2 (E(x))^2$

3rd Assignment.

20811A0432

1) a) state and prove the properties of Joint distrubution and Joint density function.

Statement :- If X & Y are two Random variables then $P[X \le x, Y \le y]$ is called joint distrubution function or CDF of (X,Y)

→ It is denoted as $F_{XY}(x,y) = P(X \le x, Y \le y)$

Mathematically, $F_{XY}(X,Y) = P[(X \le x, Y \le y)]$

$$= P[X \le x \cap Y \le y]$$

→ If x and y are continuous Random variable then, $F_{XY}(x,y)$

$$F_{XY}(x,y) = P[X \le x, Y \le y] = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{XY}(x,y)\,dx\,dy$$

→ If x and y are discrete random variable the distrubution function

$$F_{XY}(x,y) = P(X \le x \cdot Y \le y) = \sum_{n=1}^{N}\sum_{n=1}^{M} P(x_n, y_n)\,\mu(x-x_n)\,\mu(y-y_n)$$

properties :-

property-1 :-

$$F_{XY}(-\infty,-\infty) = 0$$

$$F_{XY}(x,-\infty) = 0$$

$$F_{XY}(-\infty,y) = 0$$

4. RANDOM PROCESS (Assignment-4) (15)

① (a) Define Autocorrelation function write and its properties of Random process?

A:- If Random process $x(t)$ is wide sense stationary (WSS) Random then Autocorrelation function of $x(t)$ has

$R_{XX}(\tau) = E[X(t)X(t+\tau)]$ for All values τ.

properties :-

1. Auto correlation function is an even function of τ

i.e $R_{XX}(-\tau) = R_{XX}(\tau)$

Explanation

$R_{XX}(\tau) = E[X(t)X(t+\tau)]$

$\tau = -\tau$

$R_{XX}(-\tau) = E[X(t)X(t-\tau)]$

$= E[X(t-\tau)X(t)]$

$= E[X(t-\tau)X(t-\tau+\tau)]$

$= E[X(u)X(u+\tau)]$ [let $t-\tau = u$]

$= R_{XX}(\tau)$

2. Mean squared value of Random process $x(t)$ is

$E[X^2(t)] = \lim_{\tau \to 0} R_{XX}(\tau)$

Explanation

$R_{XX}(\tau) = [X(t)X(t+\tau)]$

$\lim_{\tau \to 0} R_{XX}(\tau) = \lim_{\tau \to 0} E[X(t)X(t+\tau)]$

$= E[X^2(t)]$

UNIT - V

Assignment - 5

(5) 20811A0428

1a) State and Prove the relation between PDS and Auto correlation function (or) Derive the wiener - khirchin relation for ACF and PSD.

The Power spectral density function and time average of auto correlation function are form fourier transform Pairs.

i.e ; $F[A[R_{xx}(t, t+\tau)]] = S_{xx}(\omega)$ (or)

$$T|S_{xx}(\omega)| = A[R_{xx}(t, t+\tau))]$$

Proof :- consider w.s.s random process $x(t)$ the fourier transformation trenkared signal $x_T(t)$ over the interval $(-T, T)$ is given by

$$X_T(\omega) = F[x_T(t)] = \int_{-T}^{T} x(t) e^{-j\omega t} dt$$

As per defination ;

$$S_{xx}(\omega) = \lim_{T\to\infty} \frac{E[|x_T(\omega)|^2]}{2T}$$

$$= \lim_{T\to\infty} \frac{E[X_T(-\omega) X_T(\omega)]}{2T}$$

$$= \lim_{T\to\infty} \frac{E\left[\int_{-T}^{T} x(t_1) e^{-j(-\omega)t_1} dt_1 \int_{-T}^{T} x(t_2) e^{-j\omega t_2} dt_2\right]}{2T}$$

$$= \lim_{T\to\infty} \frac{\int_{-T}^{T}\int_{-T}^{T} E[x(t_1)x(t_2)] e^{-j\omega(t_2-t_1)} dt_1 dt_2}{2T}$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T}\int_{-T}^{T} R_{xx}(t_1, t_2) e^{-j\omega(t_2-t_1)} dt_1 dt_2$$

since $t_1 = t$, and $t_2 = t+\tau$, $dt_1 = dt$, $dt_2 = d\tau$

$$= \lim_{T\to\infty} \int_{-T}^{T} \left[\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} R_{xx}(t, t+\tau) dt\right] e^{-j\omega\tau} d\tau$$

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Tutorial 1

1. The random variable x has the discrete variable in the set{-1, 0.5, 0.7,1,5,3}, the corresponding probabilities are assumed to be {0.1,0.2,0.1,0.4,0.2} plot its distribution function?
2. If the probability density of a random variable is given by

$$f_x(x)=\begin{cases} c.\exp\left(-\frac{x}{4}\right) & \text{for } 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Find the value of 'c' evaluate $F_x(0.5)$?

3. Two dies are thrown. the square of the sum of the points appearing on the two dies is a random variable X. determine the values taken by X_1 and the corresponding probabilities?

4. State and prove the properties of probability density function?

5. Explain about the distribution and density functions of Rayleigh random variable with neat sketches?

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Tutorial 2

1. A random variable X has a PDF

$$F_x(x) = \begin{cases} \frac{1}{2}\cos x & \text{for } -\pi/2<x<\pi/2 \\ 0 & \text{Otherwise} \end{cases}$$

Find the mean value of the function $g(x) = 4x^2$?

2. If x is a discrete random variable with probability mass function given as below table

x	-2	-1	0	1	2
P(X)	1/5	2/5	1/10	1/10	1/5

Find 1) E[x] 2) $E[X^2]$ 3) E [2X+3] 4) $E[(2x+1)^2]$

3. State and prove properties of moment generating function?

4. Let Y=2x+3, if the random variable x is uniformly distributed over [-1, 2], determine $f_Y(y)$?

5. Show that E[X+Y] =E[X] +E[Y]?

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Tutorial 3

1. The joint density function for X and Y is

$$f_{xy}(X,Y) = \begin{cases} \frac{xy}{9} & \text{for } 0 < x < 2, 0 < y < 3 \\ 0 & otherwise \end{cases}$$

Find the conditional density function?

2. The joint density function of X and Y is given by

$$f_{xy}(x,y) = \begin{cases} a\,x^2y & \text{for } 0 < x < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

1) find 'a' show that the function is a valid density function

2) find the marginal density functions?

3)the joint PDF of a bi-variable(x,y) is given by

$$f_{xy}(x,y) = \begin{cases} k.xy & \text{for } 0 < x < y < 1 \\ 0 & otherwise \end{cases}$$

where k is a constant

1)find the value of k 2)are X and Y are independent?

4)If X and Y are independent, then show E[XY]=E[X]E[Y]?

5)Let Z is the sum of the two independent random variables X and Y find the PDF of z?

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Tutorial 4

1.consider a random process x(t)=A coswt, where w is a constant and A is a random variable uniformly distributed over(0,1).find the auto correlation and auto covariance of x(t)?

2.given E[x]=6 and $R_{xx}(t,t+\tau)=36+25\exp(-\tau)$ for a random process x(t).indicate which of the following statements are true.

1)is ergodic 2)is wide sense stationary?

3.derive an expression that relates autocorrelation function and auto covariance function?

4.what is auto correlation function,list out its properties?

5.show that $|R_{xx}(\tau)| \leq R_{xx}(0)$?

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Tutorial 5

1.find whether given power spectrum $\cos 8w/2+w^4$ is valid or not?

2.show that $S_{xx}(-w)=S_{xx}(w)$?

3.power spectrum and auto correlation functions are a fourier transform pairs. prove this statement?

4.a wss random process x(t) which has the power spectral density

$$S_{xx}(w)=\frac{w^2}{w^4+10w^2+9}$$

Find the auto correlation function and mean square value of the process?

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Tutorial 6

1. derive the expression for noise figure of two-stage cascaded network?

2. prove that $S_{yy}(\omega)=|H(\omega)|^2 S_{xx}(\omega)$?

3. list the properties narrow band random process?

4. derive the relationsip between autocorrelation of output random process of an LTI system when the input is a WSS process?

5. Find the mean square value of the output response for a system having $h(t) = e^{-t}u(t)$ and input of white noise $N_0/2$?

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NAME : K. Bhargavi

TUTORIAL-I

2) If the probability density of a random variable is given by

$$f_x(x) = \begin{cases} c.\exp(-x/4) & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of c and evaluate $F_x(0.5)$.

A:

Given data

$$f_x(x) = \begin{cases} c.\exp(-x/4) & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

By using density function property

$$\int_{-\infty}^{\infty} f_x(x)\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{0} f_x(x)\,dx + \int_{0}^{1} f_x(x)\,dx + \int_{1}^{\infty} f_x(x)\,dx = 1$$

$$\Rightarrow 0 + \int_{0}^{1} c\, e^{-x/4}\,dx + 0 = 1$$

$$= c.\int_{0}^{1} e^{-x/4}$$

$$= c.\left[\frac{e^{-x/4}}{-1/4}\right]_0^1$$

$$c\left[\frac{e^{-1/4}}{-1/4} - \frac{e^{-0}}{-1/4}\right] = 1$$

$$\Rightarrow c\left[-4(0.77-1)\right] = 1$$

$$\Rightarrow c\left[-4(-0.23)\right] = 1$$

$$\Rightarrow c\left[0.92\right] = 1$$

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SUB: RVSP

UNIT: 1

UNIT WISE QUESTIONS

1. Give example for continuous random variable and discrete random variable?
2. List any two properties of conditional density function?
3. A noisy transmission channel has a pre – digit error probability Pe=0.01. Calculate the probability of more than one error in 10 received digits?
4. Explain about the distribution and density function of exponential random variable with neat sketches?
5. If the probability density of a random variable is given by

$$f_x(X) = \begin{cases} x \; for \;\; 0 < x < 1 \\ (2 - x) \; for \; 1 < x < 2 \end{cases}$$

   Find 1) p {0.2<x<0.8}
   2) P {0.6<x<1.2}
6. Explain about distribution and density functions of a binomial random variable with neat sketches?
7. A binary source generates digits 0 & 1 randomly with probabilities 0.6 and 0.4 respectively. What is the probability that two 1's and three 0's will occur in a five-digit sequence?

   Hint: let x be the random variable denoting the number of 1's generated five-digit sequence.

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UNIT- 2

1. State Chebyshev's inequality and prove it?
2. Find the relationship between fx(X) and fy(Y) if Y=ax+b?
3. State and prove the properties of the characteristic function of a random variable?
4. What is meant by expectation? State and prove its properties?
5. Find the second central moment of a random variable with PDF

FX(X) =aexp (-ax) u(x)?

6. Write notes on monotonic transformations for a continuous random variable.

7. Let Y= x^2 find FY(Y) if x=N (0, 1)?

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UNIT-3

1. What is the probability density function of sum of two random variables?
2. Define correlation coefficient of joint random variable and marginal probability density functions?
3. Explain central limit theorem with equal and unequal distributions?
4. List all the properties of jointly Gaussian random variables?
5. Let X and Y be defined by $X=\cos\theta$ and $Y=\sin\theta$ where θ is a random variable uniformly distributed over $[0, 2\pi]$; show that X and Y are not independent?
6. Write notes on linear transformation of a Gaussian random variable.

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UNIT-4

1. Explain stationary and ergodic random process?
2. What is auto correlation and cross correlation. List out its properties?
3. Give the classification of random process?
4. Given a random process x (t) =kt, where k is a random variable uniformly distributed over $(0,2\pi)$, show that x and y are not independent?
5. State the conditions for a wss random process?
6. A random process is described by $x(t)= A^2\cos^2(w_c t+\theta)$.A and w_c are constants and θ is a random variable uniformly distributed between $\pm\pi$. is x(t) a wide sense stationary?
7. Define 1) covariance – stationary random process?

   2) Auto correlation – stationary random process?

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UNIT-5

1. If $R_{yy}(\tau)=R_{xx}(\tau)\cos(W_c\tau)$,determine $S_{yy}(w)$?
2. Find whether given power spectrum, $S_{yy}(w)=\cos^2(w)\exp(-8w^2)$ is valid or not?
3. Define cross power density spectrum and list out its properties?
4. Consider the random process $x(t)=\cos(w_o t+\theta)$ is wss.if it is assumed that w_c is a constant and θ is uniformly distributed on the interval $(0,2\pi)$?
5. The PSD of x(t) is given by $s_{xx}(w)=\begin{cases}1+w^2 & \text{for } |w|<1\\ 0 & otherwise\end{cases}$

6.show that the power spectrum of a real random process x(t) is real?

7.state and prove wiener-khinchin relation?

1. List out the properties of band limited random process.

2. Find output response of cross correlation when random process x (t) is applied to a LTI system having input response h (t)?

3. Derive the expression for effective noise temperature of a cascaded system in terms of its individual input noise temperature?

4. Write short notes on the following: Also, draw its power spectrum.

   1) Band limited white noise

   2) Thermal noise.

5. Find output response of auto correlation when random process X (t) is applied to an LTI system having input response h (t)?

6. Define generalized nyquist theorem?

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II B. Tech I Sem – ECE-I&II **SUBJECTIVE TEST – I**

SUBJECT: Random Variables and Stochastic Processes (R20)

Date :18 11 2022 **Time: 90 Min.** **Max. Marks: 30**

Answer All the following questions

01. a. **Define and Explain the following with an example**
- **i.** **Conditional probability**
- **ii.** **discrete sample space**
- **iii.** **Continuous random variable** [CO1]

b. **A Gaussian random variable 'x' has m_x=2 and σ_x =2**
- **i.** **find p(x>1.0)**
- **ii.** **find p(x≤-1.0)** [CO2]

02. a. **State and prove the Chebychev's inequality theorem** [CO1]

b. **find the moment generating function of the random variable 'x' whose moments are m_r =(r+1)!2^r** [CO2]

03. a. **State and prove the properties of joint distribution function.** [CO1]

b. **A Gaussian random variable 'x' having a mean value of zero and variance one is transformed to another new random variable 'y' by a square law transformation. Find the density function of 'y'.** [CO3]

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b. **A Gaussian random variable 'x' having a mean value of zero and variance one is transformed to another new random variable 'y' by a square law transformation. Find the density function of 'y'.**

Subjective test - 1

① a. Define and explain the following with an Example.

(i) Conditional Probability

(ii) discrete sample space

(iii) continuous random variable.

Ans:

(i) Conditional Probability:

For two events A and B then the Conditional Probability of Event A, assuming that event B has happened

i.e $P(\frac{A}{B}) = \frac{P(A \cap B)}{P(B)}$ provided $P(B) \neq 0$

Similarly the Conditional Probability of event B, assuming that Event A has happend

ie $P(B/A) = \frac{P(A \cap B)}{P(B)}$ Provided $P(A) \neq 0$

Eg:- To the fair die Experiment the Conditional probability of getting 1. Given that odd number faces is obtained.

Sol:- Sample space $S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

Event $A = \{1\}, n(A) = 1$

Event $B = \{1, 3, 5\}, n(B) = 3$

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

$A \cap B = \{1\}$, $n(A \cap B) = 1$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{6}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{6}$$

$$P(A/B) = \frac{1/6}{3/6} = \frac{1}{3}$$

(ii) Discrete Sample space:.

A sample space may contain a number of out comes that depends on the experiment If it contains a finite Number of outcomes then it is known as Discrete (or) finite sample space.

Eg:. In tossing a coin

i.e $S = \{H, T\}$

In Rolling a dice

$S = \{1, 2, 3, 4, 5, 6\}$

(iii) Continuous Random variable:.

If a Random variable takes infinite number of uncountable value its known as continuous Random variable.

Eg:. The length of Time During which vaccum tube is Installed in the circuit function i.e 0 - 120 sec.

1 b. A Gaussian random variables 'x' has $m_x = 2$ and $\sigma_x = 2$.

(i) find $P(x > 1.0)$

(ii) find $P(x \leq -1.0)$

Sol: We know that Guassian distribution of random variable is given by

$$f_X(x) = P(X \leq x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \int_{-\infty}^{x} e^{-\frac{(x-m_x)^2}{2\sigma_x^2}} dx.$$

given $m_x = 2$; $\sigma_x = 2$

it is know that

$$f_X(x) = F\left(\frac{x - m_x}{\sigma_x}\right) = F(2)$$

$$P(x \leq -1.0) = F_X(-1.0) = F\left(\frac{-1-2}{2}\right) = F(-1.5)$$

$$F(-x) = 1 - F(x) = Q(x)$$

$$F(-1.5) = 1 - F(1.5) = Q(1.5)$$

The approximation Q function is defined as $= \frac{1}{0.661x + 0.339\sqrt{x^2 + 5.51}} e^{-\frac{x^2/2}{\sqrt{2\pi}}}$

$$Q(1.5) = \frac{1}{0.661(1.5) + 0.339\sqrt{(1.5)^2 + 5.51}} e^{\frac{-x^2/2}{\sqrt{2\pi}}}$$

$$= 0.0668$$

$$\therefore P(x \leq -1.0) = F(-1.5) = Q(1.5)$$

$$= 0.0668.$$

(ii) $P(x > 1.0) = 1 - P(x \leq 1.0) = 1 - F_X(1.0)$

$$F_X(1.0) = F\left(\frac{x - m_x}{\sigma_x}\right) = F\left(\frac{1-2}{2}\right) = F(0.5)$$

$$F(-x) = 1 - F(x) = Q(x)$$

$$\Rightarrow F(-0.5) = 1 - F(0.5) = Q(0.5)$$

$$Q(x) = \frac{1}{0.661x + 0.399\sqrt{x^2 + 5.51}} \, e^{\frac{-x^2/2}{\sqrt{2\pi}}}$$

$$= \frac{1}{0.661(0.5) + 0.339\sqrt{(0.5)^2 + 5.51}} \, e^{\frac{-(0.5)^2/2}{\sqrt{2\pi}}}$$

$$= 0.3085$$

$$P(X > 0.1) = 1 - F_X(1.0)$$

$$= F(0.5)$$

$$= 1 - Q(0.5)$$

$$= 1 - 0.3085$$

$$= 0.6915$$

② a) State and prove the chebychev's inequality theorem.

Ans:- Chebychev's inequality theorm:-

Theorm:- If 'x' is a given Random variable with mean value $\bar{x}$ (or) m and variance "σ" it states that

$$\Rightarrow P\{|x - m| \geq c\} \leq \frac{\sigma^2}{c^2}$$

Where c is any small (+ve) small

<u>Proof</u> :. $\sigma^2 = E\left[(x-m)^2\right] = \int_{-\infty}^{\infty} (x-m)^2 f_x(x)\, dx$

$$\sigma^2 = \int_{-\infty}^{m-c} (x-m)^2 f_x(x)\, dx + \int_{m-c}^{m+c} (x-m)^2 f_x(x)\, dx.$$

$$+ \int_{m+c}^{\infty} (x-m)^2 f_x(x)\, dx$$

$$\sigma^2 \geq \int_{-\infty}^{m-c} (x-m)^2 f_x(x)\, dx + \int_{m+c}^{\infty} (x-m)^2 f_x(x)\, dx.$$

in the first integeral, $x \leq m-c \Rightarrow x-m \leq -c \Rightarrow$
$\Rightarrow (x-m)^2 \geq c^2$

in the 2nd integeral, $x \geq m+c \Rightarrow x-m \geq c \Rightarrow (x-m)^2 \geq c^2$

$$\sigma^2 \geq \int_{-\infty}^{m-c} c^2 f_x(x)\, dx + \int_{m+c}^{\infty} c^2 f_x(x)\, dx$$

$$\sigma^2 \geq c^2 \left[\int_{-\infty}^{m-c} f_x(x)\, dx + \int_{m+c}^{\infty} f_x(x)\, dx\right]$$

$$\sigma^2 \geq c^2 \left[P(x \leq m-c) + P(x \geq m+c)\right]$$

$$\sigma^2 \geq c^2 \left[P\{m-c \geq x \geq m+c\}\right]$$

$$\sigma^2 \geq c^2 \left[P\{-c \geq x-m \geq c\}\right]$$

$$\sigma^2 \geq c^2 \left[P\{|x-m| \geq c\}\right]$$

$$\Rightarrow P\{|x-m| \geq c\} \leq \frac{\sigma^2}{c^2}$$

(2) b. Find the moment generating function of the random variable 'x' whose moments are $m_r = (r+1)!\,2^r$

Sol: Given r^{th} moment about the origin.

$$m_r = (r+1)!\,2^r$$

r^{th} moment about the origin

$$m_r = \frac{d^r}{dt^r} M_x(t) \Big|_{at\ t=0}$$

Moment generating function of random variable x is defined as

$$M_x(t) = E[e^{tx}]$$

$$M_x(t) = E[e^{tx}]$$

$$= E\left[1 + \frac{tx}{1!} + \frac{t^2x^2}{2!} + \frac{t^3x^3}{3!} + \cdots\right]$$

$$= 1 + \frac{t}{1!} E(x) + \frac{t^2}{2!} E(x^2) + \frac{t^3}{3!} E(x^3) + \cdots$$

$$= 1 + \frac{t}{1!} m_1 + \frac{t^2}{2!} m_2 + \frac{t^3}{3!} m_3 + \cdots$$

$$m_x(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} m_r$$

Substitute m_r in above equation.

$$M_x(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} (r+1)!\,2^r$$

$$= \sum_{r=0}^{\infty} \frac{t^r}{r!} r!\,(r+1)2^r$$

$$= \sum_{r=0}^{\infty} t^r (r+1) 2^r$$

$$= \sum_{r=0}^{\infty} (2t)^r + \sum_{r=0}^{\infty} r(2t)^r$$

in the first term.

$$\sum_{r=0}^{\infty} (2t)^r = (2t)^0 + (2t)^1 + (2t)^2 + \dots$$

Series are in geometric progression ; $r = \frac{2^{nd}\text{ term}}{1^{st}\text{ term}}$

$$\frac{a}{1-r} = \frac{1}{1-2t}$$

In the 2nd term

$$\Rightarrow \sum_{r=0}^{\infty} r(2t)^r$$

$$\frac{d}{dt}(2t)^r = r(2t)^{r-1}(2)$$

$$= 2r\frac{(2t)^r}{2t}$$

$$t\frac{d}{dt}(2t)^r = r(2t)^r$$

$$\sum_{r=0}^{\infty} r(2t)^r = \sum_{r=0}^{\infty} t \cdot \frac{d}{dt}(2t)^r$$

Change of operations.

$$= t \cdot \frac{d}{dt} \sum_{r=0}^{\infty} (2t)^r$$

$$= t \cdot \frac{d}{dt}\left(\frac{1}{1-2t}\right)$$

$$= t \cdot \frac{d}{dt}(1-2t)^{-1}$$

$$= t(-1)(1-2t)^{-1-1} \frac{d}{dt}(1-2t)$$

$$= -t(1-2t)^{-2}(-2)$$

$$= \frac{2t}{(1-2t)^2}$$

$$M_X(t) = \frac{1}{1-2t} + \frac{2t}{(1-2t)^2}$$

$$= \frac{\cancel{1+2t+2t}}{(1-2t)^2}$$

$$= \frac{1}{(1-2t)^2}$$

③ ⓐ State and prove the properties of joint distribution function.

Ans Joint distribution function :-

If X and Y are two random variables then $P(X \leq x, Y \leq y)$ is called Joint distribution function (or) Cummulative distribution function of (X, Y).

It is denoted as $F_{XY}(x, y)$

Mathematically $F_{XY}(x,y) = P(X \leq x, Y \leq y)$

$$= P(X \leq x \cap Y \leq y)$$

① If x and y are continuous random variables then.

$$F_{XY}(x,y) = P(X \geq x, Y \leq y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{XY}(x,y)\,dx\,dy$$

Where

$f_{XY}(x,y)$ = Joint density function of (X, Y).

② If x and y are discrete random variable then

$$F_{XY}(x,y) = P(X \le x, Y \le y) = \sum_{n=1}^{N} \sum_{m=1}^{M} P(x_n, y_m)\, u(x-x_n)\, u(y-y_m)$$

Properties:

Property 1:

① $F_{XY}(-\alpha, -\alpha) = 0$

② $F_{XY}(x, -\alpha) = 0$

③ $F_{XY}(-\alpha, y) = 0$

Proof.

1) As per definition.

$$F_{XY}(-\alpha, \alpha) = P(X \le -\alpha, Y \le -\alpha)$$
$$= P(X \le -\alpha \cap Y \le -\alpha)$$
$$\Rightarrow P(\phi) = 0$$
$$= 0$$

② $F_{XY}(-x, -\alpha) = P(X \le x, Y \le -\alpha)$
$$= P(X \le x \cap Y \le -\alpha)$$
$$= P(\phi)$$
$$= 0$$

since $P(X \le -\alpha) = 0$ and $P(Y \le -\alpha) = 0$

③ similarly

$$F_{XY}(-\alpha, y) = P(X \le -\alpha, Y \le y)$$
$$= P(X \le -\alpha \cap Y < y)$$
$$= P(\phi)$$
$$= 0$$

Property - 2:

① $F_{XY}(+\alpha, +\alpha) = 1$

Proof:

As per definition,

$$F_{XY}(+\alpha, +\alpha) = 1$$
$$F_{XY}(\alpha\alpha) = P(X \le \alpha, Y \le \alpha)$$
$$= P(X \le \alpha \cap Y \le \alpha)$$
$$= P(S \cap S)$$
$$= P(S) = 1$$

(3) Property :- (3) $0 \le F_{XY}(x,y) = 1$

since $F_{XY}(x,y)$ is also probability function.

Proof :-

$F_{XY}(-\alpha, \alpha) = 0$ and $F_{XY}(\alpha, \alpha) = 1$

hence $0 \le F_{XY}(x,y) \le 1$

Propary :- 4 :-

$F_{XY}(x,y)$ is monotonic and non decreasing function of two dimensional random variable (X,Y).

property 5 :-

Marginal distribution function of x and y are defined as

$$F_{XY}(x, \alpha) = F_X(x)$$

&

$$F_{XY}(\alpha, y) = F_Y(y)$$

where

$F_X(x)$ and $F_Y(y)$ are marginal distribution function of x and y (or) individual distribution function.

Proof :-

$$F_{XY}(x, \alpha) = P(X \le x, Y \le \alpha)$$
$$= P(X \le x \cap Y \le \alpha)$$
$$= P(X \le x)$$
$$= F_X(x)$$

similary.

$$F_{XY}(\alpha, y) = P(X \le \alpha, Y \le y)$$
$$= P(X \le \alpha \cap Y \le y)$$
$$= P(Y \le y)$$
$$= F_Y(y)$$

Property-6 :-

If x and y are two random variables then

$P(x_1 < x \le x_2, y_1 < y \le y_2)$

$= f_{xy}(x_1, y_1) + f_{xy}(x_2, y_2) - f_{xy}(x_1, y_2) - f_{xy}(x_2, y_1)$

Explanation :-

$(x_1 < x \le x_2, y_1 < y < y_2) = (x \le x_2, y_1 < y_2 \ldots x < x_1, y_1 < y < y_2)$

$= (x \le x_2, (y \le y_2 - y < y_1) - x \le x_1, (y \le y_2 - y \le y_2))$

$= [x \le x_2, y \le y_2 - x \le x_2, y \le y_1 - x \le x_1, y \le y_2 + x \le x_1, y \le y_1)$

$P(x_1 < x < x_2, y_1 < y \le y_2) = P(x \le x_2, y \le y_2) - P(x \le x_2, y \le y_1) - P(x \le x_1, y \le y_2) + P(x \le x_1, y < y_1)$

$= f_{xy}(x_2, y_2) - f_{xy}(x_2, y_1) - f_{xy}(x_1, y_2) + f_{xy}(x_1, y_1)$

(3) b.. A Gaussian random variable 'x' having a mean value of zero and variance one is transformed to another new random variable 'y' by a square law transformation. Find the density function of 'y'.

Sol Given data

$x \sim N(m, \sigma^2) = N(0, 1)$

Gaussian density function of random variable x is

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-m)^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad (\because m = 0, \text{ and } \sigma = 1)$$

Square law transformation.

$Y = x^2$ has know real roots for $y \leq 0$ so

$f_Y(y) = 0$ for $y < 0$

$y = x^2$ has real roots for $y > 0$ so,

$x^2 = y$

$x = \pm\sqrt{y}$ so,

$x_1 = -\sqrt{y}$ and $x_2 = \sqrt{y}$

$$\frac{dx_1}{dy} = \frac{d}{dy}(-\sqrt{y}) = \frac{-1}{2\sqrt{y}}$$

$$\frac{dx^2}{dy} = \frac{d}{dy}(\sqrt{y}) = \frac{1}{2\sqrt{y}}$$

The density function of random variable Y is given by

$$f_Y(y) = f_X(x_1)\left|\frac{dx_1}{dy}\right| + f_X(x_2)\left|\frac{dx_2}{dy}\right|$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{y}{2}} \left|\frac{-1}{2\sqrt{y}}\right| + \frac{1}{\sqrt{2\pi}} e^{-y/2} \left|\frac{1}{2\sqrt{y}}\right|$$

$$= \frac{1}{\sqrt{2\pi}} e^{-y/2} \left[\frac{1}{2\sqrt{y}} + \frac{1}{2\sqrt{y}}\right]$$

$$= \frac{1}{\sqrt{2\pi y}} e^{-y/2}$$

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II B. Tech I Sem – ECE-I&II SUBJECTIVE TEST – II

SUBJECT: Random Variables and Stochastic Processes (R20)

Date : 11 02 2022 Time: 90 Min. Max. Marks: 30

Answer All the following questions

01. a. Consider random Variables Y_1 and Y_2 related to orbitrary random variables X and Y by the co-ordinate rotation $Y_1=X\cos\theta+Y\sin\theta$, $Y_2=-X\sin\theta+Y\cos\theta$. [CO5]
i. find the Covariance of Y_1 and Y_2 (ii) for what value of θ, the random variables Y_1 and Y_2 are uncorrelated [05 Marks]

b. State and prove Autocorrelation function and its properties [CO1] [05 Marks]

02. a. Given that the autocorrelation function for a stationary ergodic process with no periodic component is [CO5] $R(\tau)=25+\frac{4}{1+6\tau^2}$ Find the mean and variance of random process x(t). [05 Marks]

b. Explain stationary process. [05 Marks] [CO4]

03. a. Derive the relationship between time average of auto Correlation function and power spectral density function. [05 Marks] [CO6]

b. Consider a random process $x(t) = \cos(wt+\theta)$ where "W" is a real constant and θ is a uniform random variable in the interval $(0, \frac{\pi}{2})$ (i) show that is not WSS process (ii) also find the average power in the process [CO5]

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01. a. Consider random Variables Y_1 and Y_2 related to orbitrary random variables X and Y by the co-ordinate rotation $Y_1=X\cos\theta+Y\sin\theta$, $Y_2=-X\sin\theta+Y\cos\theta$.
i. find the Covariance of Y_1 and Y_2 (ii) for what value of θ, the random variables Y_1 and Y_2 are uncorrelated [05 Marks]

b. State and prove Autocorrelation function and its properties [05 Marks]

02. a. Given that the autocorrelation function for a stationary ergodic process with no periodic component is $R(\tau)=25+\frac{4}{1+6\tau^2}$ Find the mean and variance of random process x(t). [05 Marks]

b. Explain stationary process. [05 Marks]

03. a. Derive the relationship between time average of auto Correlation function and power spectral density function. [05 Marks]

b. Consider a random process $x(t) = \cos(wt+\theta)$ where "W" is a real constant and θ is a uniform random variable in the interval $(0, \frac{\pi}{2})$ (i) show that is not WSS process (ii) also find the average power in the process

Subjective Test-II

① a Consider random variables Y_1 and Y_2 related to arbitary random variables X & Y by the co-ordinate rotation $Y_1 = X\cos\theta + Y\sin\theta$, $Y_2 = X\sin\theta + Y\cos\theta$ ...(1). Find the covariance of Y_1 & Y_2 (ii) For what value of θ, then the random variables Y_1 & Y_2 are uncorrelated.

Ans Given $Y_1 = X\cos\theta + Y\sin\theta$

$Y_2 = -X\sin\theta + Y\cos\theta$

i) $\text{cov}(Y_1, Y_2) = (Y_1 - \bar{Y}_1)(Y_2 - \bar{Y}_2)$

$\bar{Y}_1 = E[Y_1] = E[X\cos\theta + Y\sin\theta]$

$= E[X]\cos\theta + E[Y]\sin\theta$

$= \bar{X}\cos\theta + \bar{Y}\sin\theta$

$\bar{Y}_2 = E[Y_2] = E[-X\sin\theta + Y\cos\theta]$

$= E[-X\sin\theta] + E[Y\cos\theta]$

$= -E[X]\sin\theta + E[Y]\cos\theta$

$= -\bar{X}\sin\theta + \bar{Y}\cos\theta$

$\text{cov}(Y_1, Y_2) = (Y_1 - \bar{Y}_1)(Y_2 - \bar{Y}_2)$

$= [X\cos\theta + Y\sin\theta) - (\bar{X}\cos\theta + \bar{Y}\sin\theta)][(-X\sin\theta + Y\cos\theta) - (-\bar{X}\sin\theta + \bar{Y}\cos\theta)]$

$= [X\cos\theta + Y\sin\theta - \bar{X}\cos\theta - \bar{Y}\sin\theta][-X\sin\theta + Y\cos\theta + \bar{X}\sin\theta - \bar{Y}\cos\theta]$

$= [(X - \bar{X})\cos\theta + (Y - \bar{Y})\sin\theta][(-(X - \bar{X})\sin\theta + (Y - \bar{Y})\cos\theta]$

$$= (X-\bar{X})^2 \cos\theta\sin\theta + (X-\bar{X})(Y-\bar{Y})\sin^2\theta\cos\theta(Y-\bar{Y})(X-\bar{X})\cos^2\theta + (Y-\bar{Y})^2\cos\theta\sin\theta.$$

$$= -\frac{(X-\bar{X})^2}{2}2\cos\theta\sin\theta + (X-\bar{X})(Y-\bar{Y})[\sin^2\theta - \cos^2\theta] + \frac{(Y-\bar{Y})^2}{2}2\cos\theta\sin\theta$$

$$= -\frac{(X-\bar{X})^2}{2}\sin 2\theta + (X-\bar{X})(Y-\bar{Y})[\sin^2\theta - \cos^2\theta] + \frac{(Y-\bar{Y})^2}{2}\sin 2\theta$$

$$= \left[\frac{\sigma_y^2 - \sigma_x^2}{2}\right]\sin 2\theta + \mathrm{cov}(X,Y)\cos 2\theta = 0$$

(ii) $$\left[\frac{\sigma_y^2 - \sigma_x^2}{2}\right]\sin 2\theta = -\mathrm{cov}(X,Y)\cos 2\theta.$$

$$\frac{\sin 2\theta}{\cos 2\theta} = -\frac{2\mathrm{cov}(X,Y)}{\sigma_y^2 - \sigma_x^2}$$

$$\tan 2\theta = \frac{-2\rho\sigma_x\sigma_y}{\sigma_y^2 - \sigma_x^2}$$

$$2\theta = \tan^{-1}\left(\frac{-2\rho\sigma_x\sigma_y}{\sigma_y^2 - \sigma_x^2}\right)$$

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{-2\rho\sigma_x\sigma_y}{\sigma_y^2 - \sigma_x^2}\right)$$

1b State and prove Autocorrelation function and its properties.

Ans Auto correlation function.

Consider a random variable $X(t)$ is a stationary process of SSS or WSS then it is called auto correlation function.

$$R_{xx}(\tau) = E[x(t)\, x(t+\tau)]$$

where τ is the time difference

$$\tau = t_2 - t_1$$

Properties :-

Auto correlation function $R_{xx}(\tau)$ is an then function

Proof:- $R_{xx}(-\tau) = R_{xx}(\tau)$

$$R_{xx}(\tau) = E[x(t)\, x(t+\tau)]$$

Put $\tau = -\tau$

$$R_{xx}(-\tau) = E[x(t)\, x(t-\tau)]$$

$$= E[x(t-\tau)\, x(t+\tau-\tau)]$$

$$= E[x(u)\, x(u+\tau)] \quad [\text{where } t-\tau = u]$$

2) The mean square value is zero.

i.e $E[x^2(t)] = R_{xx}(0)$

Proof :-

$$R_{xx}(\tau) = E[x(t)\, x(t+\tau)]$$

Put $\tau = 0$

$$R_{xx}(0) = E[x(t)\, x(t+0)]$$

$$= E[x(t)\, x(t)]$$

$$= E[x^2(t)]$$

3) The auto correlation function is maximum at $\tau = 0$ (origin)

$$|E[x^2(t)]| \geq R_{xx}(\tau)$$

Proof :- consider the inequatity

$$[x(t_1) \pm x(t_2)]^2 \geq 0$$

$$E[X(t_1) \pm X(t_2)]^2 \geq 0$$

Here the two random variables $x(t_1)$ and $x(t_1)$ are with same random variables with various times t_1 and t_2

$$E[x^2(t_1) + x^2(t_2) \pm 2x(t_1)x(t_2)] \geq 0$$

$$E[x^2(t_1)] + E[x^2(t_2)] \pm 2E[X(t_1)X(t_2)] \geq 0$$

$$R_{xx}(0) + R_{xx}(0) \pm 2E[|x(t)|^2] \geq 0$$

$$\cancel{2R_{xx}(0)} \pm$$

$$\pm 2E[x(t)]^2 \leq 2R_{xx}(0)$$

$$\pm 2E[x(t)]^2 \leq R_{xx}(0)$$

$$|E[x(t)]|^2 \leq R_{xx}(0)$$

4. If the auto correlation function at $\tau = 0$ is continuous then it is continuous at every other point.

Proof :-

$$R(\tau+h) - R(\tau) = E[x(t)x(t+\tau+h)] - E[x(t)x(t+\tau)]$$

$$= E[x(t)x(t+\tau+h) - x(t)x(t+\tau)]$$

$$= E[x(t)\{x(t+\tau+h) - x(t+\tau)]$$

$$\lim_{\tau \to 0} = \{x(t+\tau+h) - x(t+\tau)]$$

$$= 0$$

If the L.S of eqn $= 0$

the R.S of eqn $= 0$

5. The auto correlation function $R_{xx}(\tau)$ is time dependent with no periodic component.

$$\lim_{\tau \to \alpha} |E(x^{\tau})|^2 = R_{xx}(\tau)$$

Proof :-

$$R_{xx}(t_1, t_2) = E[X(t_1) X(t_2)]$$

where $t_1 = t_2$

where $x(t_1)$ & $x(t_2)$ are independent

$$R_{xx}(t_1, t_2) = E[X(t_1)] E[X(t_2)]$$

$$= E[X(t)]^2$$

6. If the mean value is zero then the ergodic mean value is zero with no periodic component.

$$= E[X(t)]^2$$

$$= 0^2$$

$$= 0$$

7. $X(t \pm T_0) = x(t)$

$$x(t + \tau \pm T_0) = X(t \pm \tau)$$

proof :-

$$= E[X(t) x(t+\tau)]$$

$$= E[X(t) x(t+\tau+T_0)]$$

$$= E[X(t) x(t+\tau)]$$

$$= R_{xx}(\tau)$$

8. If the autocorrelation function has dc component K_1 then the

$$Y(t) = K + x(t)$$

$$R_{yy}(t) = K^2 + R_{xx}(t)$$

proof :- $R_{YY}(\tau) = E[Y(t)\,Y(t+\tau)]$

$= E[(K+x(t))(K+x(t+\tau))]$

$= E[K^2 + Kx(t+\tau) + x(t)x(t+\tau) + Kx(t)]$

$= E[K^2] + E[Kx(t+\tau)] + E[x(t)x(t+\tau)] + E[Kx(t)]$

$= K^2 + 0 + 0 + R_{XX}(\tau)$

$= K^2 + R_{XX}(\tau)$

9. If the auto correlation function has two random variables $x(t)$ and $y(t)$.

$Z(t) = x(t) + y(t)$

$R_{ZZ}(\tau) = E[(x(t)x(t+\tau) + (y(t)y(t+\tau)]$

$E[x(t)y(t)] + E[x(t)y(t+\tau)] + E[y(t)x(t+\tau)$

$+ E[y(t)y(t+\tau)]$

$R_{XX}(\tau) + R_{XY}(\tau) + R_{YX}(\tau) + R_{YY}(\tau)$

2 a Given that the autocorrelation function for a stationary ergodic process with no periodic component is $R(\tau) = 25 + \frac{4}{1+6\tau^2}$. Find the mean and variance of random process $x(t)$

Ans Given

auto correlation function

$R(\tau) = 25 + \frac{4}{1+6\tau^2}$

$\lim_{\tau \to 0} E[x^2(t)] = R(\tau) = 25 + \frac{4}{1+0}$

$= 25 + 4$

$E(X^2(t)) = 29$

auto correlation function for stationary ergodic process with no periodic componet. is

$$\lim_{\tau \to \infty} = E[X(t)]^2 = R(\infty) = 25 + \frac{4}{1+\infty}$$

$$= 25 + 0$$

$$E[X(t)]^2 = 25$$

$$E[X(t)] = \sqrt{25}$$

$$E[X(t)] = 5$$

The mean value $E[X(t)] = 5$

Variance of $X = E[X^2(t)] - [E[X(t)]$

$$= 29 - 25$$

$$= 4$$

2b Explain stationary process

Ans Stationary process :-

If a random variable is said to be stationary process, then the statisticed properties such mean variance does not change with respect to time.

The stationary process are divided into three types.

1) SSS process

2) Wide sense stationary process (WSS)

3) Jointly wide sense stationary process SSS Process

If the random variable $X(t)$ is invariant of dimensional conditions or under transtation of time

where $(t_1, t_2 \cdots t_n) = (t_1+h, t_2+h, \cdots t_n+h)$

where $x(t)$ = first order stationary process:-

If the random variable $x(t)$ is in the demensional condition, or under transtation of time is $x(t) = x(t+h)$

$$f(x,t) = f(x, t+h)$$

It is only applicable for first order stationary process where $x(t)$ is constant.

<u>Second order stationary process:-</u>

If the random variable is in demensional conditions (or) under transfation of time.

$$f(x, t_1 t_1) = f(x, t_1+h, t_2+h)$$

wide sense stationary process

the mean value should be constant $E[x(t)] = 0$

the auto correlation function is a function of τ

$$E[x(t)\, x(t+\tau)]$$

Join wide sense stationary process :-

Consider the random variable $x(t)$ and $y(t)$

$$R_{xx}(\tau) = E[x(t)\, x(t+\tau)]$$

$$R_{yy}(\tau) = E[y(t)\, y(t+\tau)]$$

3. a Derive the relationship between time average of auto correlation function and power spectral density function

Ans

Statement :- The relationship between time average of auto correlation and power spectral density function

$$F[A R_{xx}(t_1, t+\tau)]$$

Proof :-

$$X_T(\omega) = \int_{-\infty}^{\infty} x_T(t) e^{-j\omega t} dt$$

$$= \int_{-T}^{T} x(t) e^{-j\omega t} dt$$

$$S_{xx}(\omega) = \frac{E[|X_T(\omega)|^2]}{2T} = \frac{E[X_T(-\omega) X_T(\omega)]}{2T}$$

$$\frac{1}{2T} \int_{-T}^{T} \int_{-T}^{T} x(t_1) e^{-j\omega t_1} dt_1 \, x(t_2) e^{-j\omega t_2} dt_2$$

$$\lim_{T \to \infty} \int_{-\infty}^{\infty} \left[\frac{1}{2T} \int_{-T}^{T} x(t_1) x(t_2) e^{-j\omega(t_2 - t_1)} dt \, d\tau \right.$$

$$\int_{-\infty}^{\infty} \frac{1}{2T} \int_{-T}^{T} R_{xx}(t_1, t_2) e^{-j\omega(t_2 - t_1)} dt \, d\tau$$

$t_1 = t$. $dt_2 = d\tau$, $t_2 - t_1 = \tau$

$dt_1 = dt$ $d_2 = \tau$

$$\int_{-\infty}^{T} \frac{1}{2T} \int_{-T}^{T} R_{xx}(t, t+\tau) e^{-j\omega\tau} dt \, d\tau$$

$$\int_{-\infty}^{\infty} \left[\frac{1}{2T} \int_{-T}^{T} R_{xx}(t, t+\tau) dt \right] e^{-j\omega\tau} d\tau$$

$$\int_{-\infty}^{\infty} A\left[R_{xx}(t, t_1+\tau)\right] e^{-j\omega\tau} d\tau$$

$$FA\left[A R_{xx}(t, t_1+\tau)\right] e^{-j\omega\tau} d\tau$$

8 b Consider a random process $x(t) = \cos(\omega t+\theta)$ where "ω" is a real constant and θ is a uniform random variable in the interval $(0, \frac{\pi}{2})$ (i) Show that is not WSS process (ii) also find the average power in the process.

Ans $X(t) = \cos(\omega t+\theta)$

'ω' is a real constant and θ is a function variable in the interval $(0, \pi/2)$

$$f_\theta(\theta) = \frac{1}{b-a} \quad a \le \theta \le b$$

$$\frac{1}{\frac{\pi}{2}-0} \quad 0 \le \theta \le \frac{\pi}{2}$$

$$f_\theta(\theta) = \frac{2}{\pi}$$

(i) Mean should not be a constant

$$E[X(t)] = E[\cos(\omega t+\theta)]$$

$$\int_0^{\pi} \cos(\omega t+\theta) f_\theta(\theta) d\theta$$

$$\frac{2}{\pi}\int_0^{\pi} \cos(\omega t+\theta) d\theta$$

$$\frac{2}{\pi}\left[\sin(\omega t+\theta)\right]_0^{\pi/2}$$

$$\frac{2}{\pi}\left[\sin(\omega t+\frac{\pi}{2}) - \sin(\omega t+0)\right]$$

$$\frac{2}{\pi}\left[\cos\omega t - \sin\omega t\right]$$

Hence, this random process $x(t)$ is not a WSS process because mean value is not equal to constant

(ii) average power

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} E[x^2(t)]\,dt$$

$$E[x^2(t)] = E[\cos^2(\omega t+\theta)]$$

$$E\left[\frac{1+\cos(2\omega t+2\theta)}{2}\right]$$

$$E[1/2] + E\left[\frac{1}{2}\right]\cos(2\omega t+2\theta)]$$

$$\frac{1}{2} + \frac{1}{2}E[\cos(2\omega t+2\theta)]$$

$$\int_0^{\pi/2} \cos(2\omega t+2\theta)\,d\theta$$

$$\left[\frac{\sin(2\omega t+2\theta)}{2}\right]_0^{\pi/2}$$

$$\frac{1}{2}\left[\sin(2\omega + 2\pi/2) - \sin(2\omega t+0)\right]$$

$$\frac{1}{2}\left[-\sin 2\omega t - \sin 2\omega t\right.$$

$$\frac{2}{2\pi}\left[-\sin 2\omega t\right.$$

$$-\frac{1}{\pi}\sin 2\omega t$$

$$-\frac{1}{\pi}\sin 2\omega t$$

$$\frac{1}{2} - \frac{1}{\pi}\sin 2\omega t$$

$$\frac{1}{2T}\int_{-T}^{T} \frac{1}{2} - \frac{1}{\pi}\sin 2\omega t\,dt$$

Code No: R2021044 **R20** SET - 1

II B. Tech I Semester Regular Examinations, Feb/March - 2022
RANDAM VARIABLES AND STOCHASTIC PROCESSES
(Com to ECE, ECT)

Time: 3 hours Max. Marks: 70

Answer any **FIVE** Questions each Question from each unit
All Questions carry **Equal** Marks
~~~~~~~~~~~~~~~~~~~~~~~~~~

1 a) State and prove the properties of cumulative distribution function (CDF) of X. (CO1) [7M]

b) If the communicative distribution function of a random variable X is given by [7M]

$$f_X(x) = \begin{cases} \frac{x^2}{3} & -1 \le x, y \le 2 \\ 0 & Else\ w\ \ ere \end{cases}$$

Find P(0<X<1) and $F_x(x)$? (CO2)

Or

2 a) Define conditional probability distribution function and write the properties. (CO1) [7M]

b) Two boxes are selected randomly. The first box contains 2 white balls and 3 black balls. The second box contains 3 white and 4 black balls. What is the probability of drawing a white ball? [7M] (CO2)

3 a) State and prove the Chebychev's inequality theorem. [CO1] [7M]

b) A Gaussian random variable with variance 10 and mean 5 is transformed to $y = e^x$. Find the pdf of y. [7M] [CO3]

Or

4 a) Show that any characteristic function $\Phi_X(\omega)$satisfies $\Phi_X(\omega) \le \Phi_X(0) = 1$. [CO2] [7M]

b) A random variable X is defined by density function [7M]

$$f_X(x) = \begin{cases} 3x^2 & for\ 0 \le x \le 1 \\ 0 & elsew\ \ ere \end{cases}$$ [CO2]

Find E[X], E[3X] and E[X^2].

5 a) Explain central limit theorem with equal and unequal distributions. [7M]

b) If X and Y are independent, show that E[XY]=E[X] E[Y]. [CO3] [7M]

Or

6 a) Two statistically independent random variables X and Y have respective densities $f_X(x) = 5e^{-5x}u(x), f_Y(y) = 2e^{-2y}u(y)$. Find the density of the sum W = X +Y. [7M] [CO3]

b) Gaussian random variables X and Y have first and second order moments m_{10}=-1.1, m_{20}=1.16, m_{01}=1.5, m_{02} =2.89, R_{XY}=-1.724. Find C_{XY}, ρ. [7M] [CO4]

7 a) Derive the relation between correlation and covariance of two random variables X and Y. [7M] [CO3]

b) A random process $X(t) = A\,Cos(\omega_c t + \theta)$ where θ is a random variable uniformly distributed in the range$(0,2\pi)$. Show that the process is ergodic in mean and correlation sense. [7M] [CO4]

Or
~~~~~~~~~~~~~~~~~~~~~~~~~~

8 a) The auto correlation function for a stationary ergodic process with no periodic components is $R_{XX}(\tau) = 625 + \frac{16}{1+36\tau^2}$ .Find mean and variance of the random process. [CO4] [7M]

b) Explain about Poisson random processes. [CO4] [7M]

9 a) Derive the relationship between cross-power spectral density and cross correlation function. [CO5] [7M]

b) Define the following systems. [7M]
(i)Band pass process
(ii) Band –Limited process
(iii)Narrow band process
(iv)Band – Limited Band pass process [CO1]

Or

10 a) Show that $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$. [CO5] [7M]

b) Find the mean and mean- squave values of output y(t) of an LTI system with input x(t). Assume that x(t) is a WSS process. [CO6] [7M]

Code No: R1921044 **R19** **SET - 1**

II B. Tech I Semester Regular Examinations, March - 2021
RANDOM VARIABLES AND STOCHASTIC PROCESSES
(Electronics and Communication Engineering)

Time: 3 hours Max. Marks: 75

Answer any FIVE Questions each Question from each unit
All Questions carry Equal Marks

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1 a) Define conditional probability distribution function and write he properties [CO1] [8M]

b) A random variable X is defined by [7M]

$$X(i)=\begin{cases}-2 & i\le -2\\ i & -2<i\le 1\\ 1 & 1<i\le 4\\ 6 & 4<i\end{cases}$$

Show, by a sketch, the value x into which the values of i are mapped by x. [CO1]
What type of random variable is X?

Or

2 a) Given that a random variable X has the following possible values, state if X is discrete, continuous or mixed [8M]

i. {-20<x<-5}
ii. {10,12<x<=14,15,17)
iii. {-10 for s>2 and 5 for s<=2, where 1<s<=6}
iv. {4,3.1,1, -2}

[CO1]

b) Suppose height to the bottom of clouds is a Gaussian random variable for which ax=4000m and σx=1000m. A person bets that cloud height tomorrow will fall in the set A={1000m<X≤3000m} while a second person bets that height will be satisfied by B={2000m<X≤4200m}.A third person bets they are both correct. Find the probability that each person will win the bet. [7M] [CO1]

3 a) The random variable X has characteristics function $\phi_X(w)=[a/a-jw]^N$ for a>0 and N=1,2,3..... Show that $\bar{X}=N/a$, $\bar{X}^2 N(N+1)/a^2$, and $\sigma_x^2=N/a^2$. [8M] [CO2]

b) Find mean and variance of Gaussian random variable? [7M] [CO2]

Or

4 a) A random variable X is uniformly distributed on the interval (-5,15).Another random variable $Y=e^{(-\frac{X}{5})}$ is formed. Find E[Y]. [8M] [CO2]

b) A Gaussian voltage random variable X has a mean value ax=0 and σ^2x=9. The voltage X is applied to a square-law, full wave diode detector with a transfer characteristics $Y=5X^2$. Find the mean value of the output voltage Y. [7M] [CO2]

5 a) Random variable X and Y have the joint density [8M]

$$F_{X,Y}(x,y)=\begin{cases}1/24 & 0<x<6 \text{ and } 0<y<4\\ 0 & \text{elsewhere.}\end{cases}$$

[CO3]

What is the expected value of the function $g(X,Y)=(XY)^2$?

b) Two statistically independent random variable X and Y have mean values $\bar{X}$ =E[X]=2 and E[Y]=4. They have second moments $\bar{X}^2$=E[X^2]=8 and E[Y^2]=25. Find i) the mean value ii) the second moment iii) the variance of the random variable W=3X-Y. [7M] [CO3]

Or

||"|"||"||""""|

6 a) For the two random variable X and Y: [8M]

$$F_{X,Y}(x,y)=0.15\delta(x+1)\delta(y)+0.1\delta(x)\delta(y)+0.1\delta(x)\delta(y-2)+0.4\delta(x-1)\delta(y+2)$$
$$+0.2\delta(x-1)\delta(y-1)+0.5\delta(x-1)\delta(y-3)$$ [CO3]

Find: i) the correlation, ii) the covariance, iii) the correlation coefficient of X and Y and iv) are X and Y either uncorrelated or orthogonal?

b) Gaussian random variable X_1 and X_2 for which $\bar{X}_1=2, \sigma_{X1}^2=9, \bar{X}_2=-1, \sigma_X^2=4$ and $C_{X1X2}=-3$ are transformed to new random variable Y_1 and Y_2 according to $Y_1=-X_1+X_2, Y_2=-2X_1-3X_2$. Find [7M]
i) σ_{Y1}^2 ii) σ_{Y2}^2 iii) C_{Y1Y2}. [CO3]

7 a) Let X (*t*) be a stationary continuous random process that is differentiable. Denote its time derivative by $\dot{X}(t)$. Show that $E\left(\dot{X}(t)\right)=0$. [CO4] [8M]

b) Given the random process by $X(t)=A\cos(w_0t)+B\sin(w_0t)$ [7M]
Where w_0 is a constant, and A and B are uncorrelated zero mean random variables having different density functions but the same variance ,show that X(t) is wide sense stationary but not strictly stationary. [CO4]

Or

8 a) A random process is defined by X(*t*) = A, where A is a continuous random variable uniformly distributed on (0, 1). Determine the form of the sample functions, classify the process [CO4] [8M]

b) Define ergodic random proven? Explain with example. [CO4] [7M]

9 a) Drive the Wiener-Khintchine relation. [CO5] [10M]

b) What is Mean value of System Response for Random Signal Response of Linear Systems. [CO6] [5M]

Or

10 A Random signal X(t) of PSD of $\frac{N_0}{2}$ is applied on an LTI system having impulse response $h(t)$. If $Y(t)$is output, find (i) $E[Y^2(t)]$ (ii) $R_{XY}(\tau)$ (iii) $R_{YX}(\tau)$ (iv) $R_{YY}(\tau)$. [CO6] [15M]

ANTHI INSTITUTE OF ENGINEERING & TECHNOLOGY
MAKAVARAPALEM MANDAL, VISAKHA DIST.

II B.TECH. - I SEM ECE-I (R20) (2020 ADMITTED BATCH) MID & ON - LINE MARKS KEY LISTS FOR THE YEAR 2021 -22

Name of the Faculty : R. PRASAD RAO							Subject Code		R2021044		Name of the Subject				RANDOM VARIABLES AND STOCHASTIC PROCESSES				
S.NO.	ROLL NO.	NAME OF THE STUDENT	I Mid (30 Marks)	I Mid (15 Marks) (A)	I Objective (20 Marks)	I Objective (10 Marks) (B)	Assignment Marks (05 M) (C)	Test I Marks (A+B+C)	II Mid (30 Marks)	II Mid (15 Marks) (D)	II Objective (20 Marks)	II Objective (10 Marks) (E)	Assignment Marks (05 M) (F)	Test II Marks (D+E+F)	Best Test (80%) (G)	Non Best Test (20%) (H)	Final Marks (30 Marks) (G+H)	Test - I Analysis	
1	20811A0401	ADAPUREDDI HEMA KIRAN	30	15	4	2	5	22	30	15	12	06	5	26	20.8	4.4	26	Range	Students
2	20811A0402	ALAMANDA GIRIDHAR SHANKAR SAI PRAKASH	AB	AB	AB	AB	5	5	AB	AB	AB	AB	AB	AB	AB	AB	AB	<10	11
3	20811A0403	ANNAMREDDY RENUKA	29	15	8	4	5	24	28	14	15	08	5	27	21.6	4.8	27	11-15	3
4	20811A0404	ANTHONY RAJ JUSTINA	25	13	5	3	5	21	17	09	05	03	5	17	16.8	3.4	21	16-19	9
5	20811A0405	AZAD TEPPALA	16	8	AB	AB	5	13	27	14	05	03	5	22	17.6	2.6	21	20-24	19
6	20811A0406	BANDARU GANGADHAR	20	10	6	3	5	18	23	12	03	02	5	19	15.2	3.6	19	25-30	2
7	20811A0407	BANDARU HARIKA	30	15	10	5	5	25	24	14	08	04	5	23	20.0	4.6	25	Absenties	3
8	20811A0408	BANDARU UDAY KAMAL	13	7	6	3	5	15	18	09	03	02	5	16	12.8	3.0	16	Total	49
9	20811A0409	BANTUBILLI SIRISHA	28	14	4	2	5	21	20	10	09	05	5	20	16.8	4.0	21		
10	20811A0410	BARNIKALA LOKESH	21	11	4	2	5	18	26	13	08	04	5	22	17.6	3.6	22	Best Test Analysis	
11	20811A0411	BEERA MAHESH BABU	27	14	4	2	5	21	27	14	08	04	5	23	18.4	4.2	23	Range	Students
12	20811A0412	BHEEMUNI YASASWANI NAMRATHA SRI	30	15	8	4	5	24	24	12	10	05	5	22	19.2	4.4	24	<10	
13	20811A0413	BODDETI PRASANTHI	29	15	AB	AB	5	20	28	14	12	06	5	25	20.0	4.0	24	11-15	
14	20811A0414	BODDUPALLI NIKHIL TEJA	0	0	AB	AB	5	5	16	08	04	02	5	15	12.0	1.0	13	16-19	
15	20811A0415	BOTTA TULASI	29	15	8	4	5	24	26	13	11	06	5	24	19.2	4.8	24	20-24	
16	20811A0416	CHANDAKA PAVANI	25	13	7	4	5	22	23	12	05	03	5	20	17.6	4.0	22	25-30	
17	20811A0417	CHUKKALA GOWRI PARVATHI DEVI	15	8	AB	AB	5	13	20	10	03	02	5	17	13.6	2.6	17	Absenties	

S.NO.	ROLL NO.	NAME OF THE STUDENT	I Mid (30 Marks)	I Mid (15 Marks) (A)	I Objective (20 Marks)	I Objective (10 Marks) (B)	Assignment Marks (05 M) (C)	Test I Marks (A+B+C)	II Mid (30 Marks)	II Mid (15 Marks) (D)	II Objective (20 Marks)	II Objective (10 Marks) (E)	Assignment Marks (05 M) (F)	Test II Marks (D+E+F)	Best Test (80%) (G)	Non Best Test (20%) (H)	Final Marks (30 Marks) (G+H)	Test - I Analysis
18	20811A0418	EARLA GURU PRATHAP	3	2	AB	AB	5	7	21	11	05	03	5	19	15.2	1.4	17	Total
19	20811A0419	GANDROTHU VENKATA SRI HARSHA	0	0	AB	AB	5	5	20	10	04	02	5	17	13.6	1.0	15	
20	20811A0420	GEMBALI LAHARI	24	12	8	4	5	21	24	12	07	04	5	21	16.8	4.2	21	
21	20811A0421	GIRIJALA VENU MADHAV	4	2	AB	AB	5	7	21	11	09	05	5	21	16.8	1.4	19	
22	20811A0422	GOKULAPATI SHYAM	AB	AB	AB	AB	5	5	07	04	03	04	5	13	10.4	1.0	12	
23	20811A0423	GOLLAPALLI DEVA MANIKANTA	14	7	5	3	5	15	14	07	06	03	5	15	12.0	3.0	15	
24	20811A0424	GONTHINA DEEPTHI	30	15	8	4	5	24	28	14	10	05	5	24	19.2	4.8	24	
25	20811A0425	GUNISETTY TANMAYEE SATYA	18	9	7	4	5	18	15	08	05	03	5	16	14.4	3.2	18	
26	20811A0426	JADDU SRIMANNARAYANA	AB	AB	AB	AB	5	5	AB	AB	07	04	5	09	7.2	1.0	09	
27	20811A0427	JAYANTHI SRINIVAS SARAT CHANDRA	18	9	3	2	5	16	11	06	04	02	5	12	12.8	3.4	16	
28	20811A0428	KETHAVARAPU BHARGAVI	29	15	5	3	5	23	29	15	09	05	5	25	20.0	4.6	25	
29	20811A0429	KILLAMPALLI PRASANNA	2	1	7	4	5	10	10	05	08	04	5	14	11.2	2.0	14	
30	20811A0430	KONNA MOHAN KARTHEEK	9	5	1	1	5	11	14	07	05	03	5	15	12.0	2.2	15	
31	20811A0431	KORATANA TEJA	17	9	7	4	5	18	21	11	05	03	5	19	15.2	3.6	19	
32	20811A0432	KORIBILLI JYOTHI	5	3	7	4	5	12	22	11	04	02	5	18	14.4	2.4	17	
33	20811A0433	KORRAYI DARAMALLESH	6	3	3	2	5	10	21	11	01	01	5	17	13.6	2.0	16	
34	20811A0434	NANDAVARAPU JAYA LAKSHMI	14	7	7	4	5	16	14	07	06	03	5	15	12.8	3.0	16	
35	20811A0435	LEKKALA KALYANI	29	15	10	5	5	25	24	12	13	07	5	24	20.0	4.8	25	
36	20811A0436	MADEM MANITEJA	28	14	4	2	5	21	23	12	07	04	5	21	16.8	4.2	21	
37	20811A0437	MALLA MONALI	23	12	7	4	5	21	25	13	03	02	5	20	16.8	4.0	21	

S.NO.	ROLL NO.	NAME OF THE STUDENT	I Mid (30 Marks)	I Mid (15 Marks) (A)	I Objective (20 Marks)	I Objective (10 Marks) (B)	Assignment Marks (05 M) (C)	Test I Marks (A+B+C)	II Mid (30 Marks)	II Mid (15 Marks) (D)	II Objective (20 Marks)	II Objective (10 Marks) (E)	Assignment Marks (05 M) (F)	Test II Marks (D+E+F)	Best Test (80%) (G)	Non Best Test (20%) (H)	Final Marks (30 Marks) (G+H)	Test - I Analysis	
38	20811A0438	MARRAPU SURESH	23	12	8	4	5	21	22	11	12	06	5	22	17.6	4.2	22		
39	20811A0439	MEDAPUREDDI SUSEELA	27	14	8	4	5	23	26	13	10	05	5	23	18.4	4.6	23		
40	20811A0440	MIDATHANA SRIVANI	29	15	5	3	5	23	30	15	10	05	5	25	20.0	4.6	25		
41	20811A0441	MOLLETI DURGA PRASAD	3	2	AB	AB	5	7	03	02	05	03	5	10	8.0	1.4	10		
42	20811A0443	MUMMINA PRAVALLIKA	10	5	6	3	5	13	14	07	03	02	5	14	11.2	2.6	14		
43	20811A0444	MUNIPALLI PAVAN KUMAR	6	3	7	4	5	12	09	05	05	03	5	13	10.4	2.4	13		
44	20811A0445	NAGIREDDY SUDHAKAR	7	4	AB	AB	5	9	14	07	03	02	5	14	11.2	1.8	13		
45	20811A0446	NAMALA TEJASWANI	16	8	7	4	5	17	19	10	08	04	5	19	15.2	3.4	19		
46	20811A0447	PADMANABHAM SAI BHARGAVI	28	14	5	3	5	22	18	09	03	02	5	16	17.6	3.2	21		
47	20811A0448	PALLELA LAVANYA	22	11	5	3	5	19	21	11	03	02	5	18	15.2	3.6	19		
48	20811A0449	PALLI SRINU	28	14	10	5	5	24	27	14	03	02	5	21	19.2	4.2	24		
49	20811A0450	PANDIRIPILLI CHANDINI	20	10	1	1	5	16	23	12	07	04	5	21	16.8	3.2	20		

	Ist Mid		2nd Mid	
Absent Roll Numbers in	20811A0402, 20811A0422, 20811A0426			
	Signatue of the Subject Faculty	Signatue of the Principal	Signatue of the Subject Faculty	Signatue of the Principal

	Best of Two		Best of Two
Roll Numbers below 10	426	Roll Numbers between 11 to 15	14, 29, 41, 43, 44, 45

AVANTHI INSTITUTE OF ENGINEERING & TECHNOLOGY
MAKAVARAPALEM MANDAL, VISAKHA DIST.

II B.TECH. - I SEM ECE-II (R20) (2020 ADMITTED BATCH) MID & ON - LINE MARKS KEY LISTS FOR THE YEAR 2021 -22

Name of the Faculty : R. PRASAD RAO | Subject Code: R2021044 | Name of the Subject: RANDOM VARIABLES AND STOCHASTIC PROCESSES

S.NO.	ROLL NO.	NAME OF THE STUDENT	I Mid (30 Marks)	I Mid (15 Marks) (A)	I Objective (20 Marks)	I Objective (10 Marks) (B)	Assignment Marks (05 M) (C)	Test I Marks (A+B+C)		II Mid (30 Marks)	II Mid (15 Marks) (D)	II Objective (20 Marks)	II Objective (10 Marks) (E)	Assignment Marks (05 M) (F)	Test II Marks (D+E+F)	Best Test (80%) (G)	Non Best Test (20%) (H)	Final Marks (30 Marks) (G+H)	Test - I Analysis	
1	20811A0451	PENTAKOTA JASHWANTH KUMAR	30	15	7	4	5	24		13	07	01	01	5	13	19.2	2.6	22	Range	Students
2	20811A0452	PENTAKOTA VASAVI	29	15	9	5	5	25		18	09	08	04	5	18	20.0	3.6	24	<10	10
3	20811A0453	PILLA PREMANJALI	27	14	8	4	5	23		23	12	06	03	5	20	18.4	4.0	23	11-15	4
4	20811A0454	PITTA NAGA DURGA PRASAD	21	11	5	3	5	19		16	08	04	02	5	15	15.2	3.0	19	16-19	12
5	20811A0455	POLUMURI APPALARAJU	24	12	6	3	5	20		17	09	05	03	5	17	16.0	3.4	20	20-25	23
6	20811A0456	POTHU LAHARI	29	15	7	4	5	24		18	09	10	05	5	19	19.2	3.8	23	26-30 Absenties	5
7	20811A0457	PULIGA SYAMA SRI	AB	AB	AB	AB	5	5		AB	AB	AB	AB	AB	AB	AB	AB	AB	Total	49
8	20811A0458	R SAI KIRAN	16	8	5	3	5	16		09	05	08	04	5	14	12.8	2.8	16		
9	20811A0459	RAPETI VARALAKSHMI	16	8	9	5	5	18		16	08	03	02	5	15	14.4	3.0	18		
10	20811A0460	REDDY DHUSYANTH	29	15	AB	AB	5	20		20	10	05	03	5	18	16.0	3.6	20	Best Test Analysis	
11	20811A0461	REVATHI MADDU	26	13	9	5	5	23		18	09	05	03	5	17	18.4	3.4	22	Range	Students
12	20811A0462	ROUTHU SWETHASRI	28	14	7	4	5	23		15	08	05	03	5	16	18.4	3.2	22	<10	
13	20811A0463	ROUTHU VENKATA DURGA PRASAD	24	12	8	4	5	21		26	13	16	08	5	26	20.8	4.2	25	11-15	
14	20811A0464	SANNIBOINA RAJESH	24	12	0	0	5	17		07	04	06	03	5	12	13.6	2.4	16	16-19	
15	20811A0465	SASAPU SATHEESH	AB	AB	AB	AB	5	5		AB	AB	AB	AB	5	05	4.0	1.0	05	20-25	
16	20811A0466	SHAIK JILANI	0	0	7	4	5	9		03	02	05	03	5	10	8.0	1.8	10	Absenties	
17	20811A0467	SINGAMPALLI RAMADINESH	AB	AB	AB	AB	5	5		AB	AB	AB	AB	5	05	4.0	1.0	05	Total	
18	20811A0468	TAMARANA HEMA ESWARI SAI KUMAR	0	0	AB	AB	5	5		08	04	07	04	5	13	10.4	1.0	12		

20 | 21815A0409 | VANNIKMA

S.NO.	ROLL NO.	NAME OF THE STUDENT	I Mid (30 Marks)	I Mid (15 Marks) (A)	I Objective (20 Marks)	I Objective (10 Marks) (B)	Assignment Marks (05 M) (C)	Test I Marks (A+B+C)		II Mid (30 Marks)	II Mid (15 Marks) (D)	II Objective (20 Marks)	II Objective (10 Marks) (E)	Assignment Marks (05 M) (F)	Test II Marks (D+E+F)	Best Test (80%) (G)	Non Best Test (20%) (H)	Final Marks (30 Marks) (G+H)	Test - I Analysis	
19	20811A0469	THURUBILLI MOHAN SAI	4	2	5	3	5	10		03	02	08	04	5	11	8.8	2.0	11		
20	20811A0470	UNDRAJAVARAPU DURGA RAO	9	5	AB	AB	5	10		07	04	06	03	5	12	9.6	2.0	12		
21	20811A0471	UPPATI DARABABU	3	2	4	2	5	9		11	06	06	03	5	14	11.2	1.8	13		
22	20811A0472	VANJARAPU THARUN KUMAR	28	14	4	2	5	21		21	11	09	05	5	21	16.8	4.2	21		
23	20811A0473	VANTEDDU BABU RAO	22	11	4	2	5	18		17	09	09	05	5	19	15.2	3.6	19		
24	20811A0474	VELAGA POOJITHA	27	14	5	3	5	22		25	13	07	04	5	22	17.6	4.4	22		
25	20811A0475	VOMMI SUMANTH	17	9	AB	AB	5	14		11	06	05	03	5	14	11.2	2.8	14		
26	20811A0476	YAKA POORNA VENKATESH	11	6	0	0	5	11		AB	AB	06	03	5	08	8.8	1.6	11		
27	20811A0477	YEDLA PUSHPA LATHA	24	12	11	6	5	23		12	06	04	02	5	13	18.4	2.6	21		
28	20811A0478	YENNI GNANESWARA RAO	15	8	7	4	5	17		21	11	02	01	5	17	13.6	3.4	17		
29	20811A0479	YERUVA BALESWAR REDDY	30	15	AB	AB	5	20		16	08	11	06	5	19	16.0	3.8	20		
30	20811A0481	KODUKULA ASWININ GANESH LIKITH KUMAR	AB	AB	AB	AB	5	5		04	02	01	01	5	08	6.4	1.0	08		
31	21815A0401	KANDREGULA GEETHA SRI LAKSHMI	16	8	8	4	5	17		26	13	10	05	5	23	18.4	3.4	22		
32	21815A0402	KUSIREDDI TULASIRAM	28	14	8	4	5	21		27	14	07	04	5	23	18.4	4.2	23		
33	21815A0403	LALAM KUSUMA NAGA GANA MALLIKA	12	6	7	4	5	15		27	14	06	03	5	22	17.6	3.0	21		
34	21815A0404	LANKA LAKSHMI SUBHA SRI HAVYA	25	13	7	4	5	22		25	13	09	05	5	23	18.4	4.4	23		
35	21815A0405	PARAVADA BHANU PRASAD	21	11	9	5	5	21		15	08	09	05	5	18	16.8	3.6	21		
36	21815A0406	POLAMARASETTI JAGADEESH	7	4	7	4	5	13		08	04	08	04	5	13	10.4	2.6	13		
37	21815A0407	PONNADA PRIYANKA	26	13	4	2	5	20		AB	AB	05	03	5	08	16.0	1.6	18		
38	21815A0408	POOLLA VENKATA SRIVANI BHARGAVI	17	9	9	5	5	19		26	13	05	03	5	21	16.8	3.8	21		
39	21815A0409	VANGURI POORNA SEKHAR	19	10	9	5	5	20		19	10	05	03	5	18	16.0	3.6	20		

S.NO.	ROLL NO.	NAME OF THE STUDENT	I Mid (30 Marks)	I Mid (15 Marks) (A)	Obje-ctive (20 Marks)	I Obje-ctive (10 Marks) (B)	Assign ment Marks (05 M) (C)	Test I Marks (A+B+C)		II Mid (30 Marks)	II Mid (15 Marks) (D)	II Obje-ctive (20 Marks)	II Obje-ctive (10 Marks) (E)	Assign ment Marks (05 M) (F)	Test II Marks (D+E+F)	Best Test (80%) (G)	Non Best Test (20%) (H)	Final Marks (30 Marks) (G+H)	Test - I Analysis	
40	21815A0410	YENUGULA MANEESHA	19	10	8	4	5	19		19	10	04	02	5	17	15.2	3.4	19		
41	21815A0411	CHEPURI HEMALATHA	20	10	6	3	5	18		25	13	07	03	5	21	16.8	3.6	21		
42	21815A0412	CHINTABATHINA NITHIN	AB	AB	AB	AB	5	5		AB	AB	04	02	5	07	5.6	1.0	07		
43	21815A0413	GARAGA SAMUEL BABU	22	11	15	8	5	24		14	07	06	03	5	15	19.2	3.0	23		
44	21815A0414	KISTAM MOUNIKA	29	15	9	5	5	25		30	15	05	03	5	23	20.0	4.6	25		
45	21815A0415	KONATHALA SASI KANTH	19	10	6	3	5	18		12	06	09	05	5	16	14.4	3.2	18		
46	21815A0416	PENTAKOTA JAYANTH	26.	13	3	2	5	20		20	10	08	04	5	19	16.0	3.8	20		
47	21815A0417	SAMMETLA VARSHINI	29	15	6	3	5	23		24	12	09	05	5	22	18.4	4.4	23		
48	21815A0418	BODDAPATI YESWANTH	22	11	4	2	5	18		26	13	06	03	5	21	16.8	3.6	21		
49	21815A0419	SETTI UMA VENKATA VARALAKSHMI	22	11	8	4	5	20		23	12	08	04	5	21	16.8	4.0	21		

	Ist Mid	2nd Mid
Absent Roll Numbers in	457, 465, 467, 481, 412	
	Signatue of the Subject Faculty / Signatue of the Principal	Signatue of the Subject Faculty / Signatue of the Principal

	Best of Two		Best of Two
Roll Numbers below 10	65, 467, 81, 642	Roll Numbers between 11 to 15	68, 69, 70, 71, 75, 76

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DEPARTMENT OF ECE

II Year I Semester

List of Weak Students

Subject: RVSP **(Candidate with Mid Marks<15)**

S.No	Roll Number	Name of the student
1.	20811A0414	B Nikhil Teja
2.	20811A0426	J Srimanarayana
3.	20811A0427	J Srinivas Chandra
4.	20811A0441	M Durga Prasad
5.	20811A0444	M Pavan Kumar
6.	20811A0445	N Sudhakar
7.	20811A0451	P Jaswanth Kumar
8.	20811A0458	R Sai Kiran
9.	20811A0464	S Rajesh
10.	20811A0465	S Satheesh
11.	20811A0466	SkJilani
12.	20811A0467	S Rama Dinesh
13.	20811A0468	T Hemaeswar Sai Kumar
14.	20811A0469	T Mohan Sai
15.	20811A0470	U Durga Rao
16.	20811A0471	U Dara Babu
17.	20811A0475	V Sumanth
18.	20811A0476	Y Poorna Venkatesh
19.	20811A0477	Y Pushpalatha
20.	20811A0481	K A G Likith Kumar
21.	21815A0406	P Jagadeesh
22.	21815A0407	P Priyanka
23.	21815A0412	C Nithin

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DEPARTMENT OF ECE

II Year I Semester

List of Weak Students

Subject: RVSP **(Candidate with Mid Marks<15)**

S.No	Roll Number	Name of the student	17/1	18/1	19/1	20/1	21/1	22/1	23/1	25/1	27/1	29/1	30/1	1/2
1	20811A0414	B Nikhil Teja	1	2	3	A	4	5	6	7	8	A	9	10
2	20811A0426	J Srimanarayana	1	2	A	3	4	5	6	7	8	9	A	10
3	20811A0427	J Srinivas Chandra	1	2	3	4	5	6	A	7	8	9	10	11
4	20811A0441	M Durga Prasad	A	1	2	3	4	5	6	7	8	9	A	10
5	20811A0444	M Pavan Kumar	1	2	3	4	5	6	7	8	A	9	10	11
6	20811A0445	N Sudhakar	1	2	3	4	5	A	6	7	8	A	9	10
7	20811A0451	P Jaswanth Kumar	1	A	2	3	4	5	A	6	7	8	A	9
8	20811A0458	R Sai Kiran	1	2	A	3	4	A	5	6	7	A	8	9
9	20811A0464	S Rajesh	1	2	3	A	4	5	6	A	7	8	9	A
10	20811A0465	S Satheesh	1	2	A	3	4	5	6	7	A	8	9	10
11	20811A0466	SkJilani	A	1	2	3	4	5	A	6	7	8	9	10
12	20811A0467	S Rama Dinesh	1	2	A	3	4	5	6	7	8	A	9	10
13	20811A0468	THemaeswarSaiKumar	1	2	3	4	5	6	A	7	8	9	10	A
14	20811A0469	T Mohan Sai	A	1	2	A	3	4	5	6	7	8	9	10
15	20811A0470	U Durga Rao	1	A	2	3	4	A	5	6	7	8	A	9
16	20811A0471	U Dara Babu	1	2	3	A	A	4	A	5	6	7	A	8
17	20811A0475	V Sumanth	1	2	A	3	4	5	6	7	8	9	A	10
18	20811A0476	Y PoornaVenkatesh	A	1	2	3	4	A	5	6	7	A	8	9
19	20811A0477	Y Pushpalatha	1	2	3	A	4	5	6	7	8	9	10	A
20	20811A0481	K A G Likith Kumar	A	1	2	3	4	5	A	6	7	8	9	10
21	21815A0406	P Jagadeesh	1	2	3	4	5	A	6	7	8	A	9	10
22	21815A0407	P Priyanka	1	2	3	A	4	5	6	A	7	8	A	9
23	21815A0412	C Nithin	A	1	2	3	4	5	A	6	7	8	A	9

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DEPARTMENT OF ECE

II Year I Semester

List of Advanced Learners

Subject: RVSP **(Candidates with Mid Marks>22)**

S.No	Roll Number	Name Of The Student
1	20811A0401	A Hema Kiran
2	20811A0403	A Renuka
3	20811A0407	B Harika
4	20811A0411	B Mahesh Babu
5	20811A0412	B Y Namratha Sri
6	20811A0413	B Prashanthi
7	20811A0415	B Tulasi
8	20811A0424	G Deepthi
9	20811A0428	K Bhargavi
10	20811A0435	L Kalyani
11	20811A0440	M Srivani
12	20811A0449	P Srinu
13	20811A0452	P Vasavi
14	20811A0453	P Premanjali
15	20811A0456	P Lahari
16	20811A0463	R V Durga Prasad
17	21815A0402	K Tulasi Ram
18	21815A0404	L L S Srihavya
19	21815A0413	G Samuel babu
20	21815A0414	K Mounika
21	21815A0417	S Varshini

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DEPARTMENT OF ECE

II Year I Semester

RESULT ANALYSIS

Number of Students Appeared: 94

Number of Students Passed: 60

Total Pass Percentage: 63.82%

AVANTHI INSTITUTE OF ENGINEERING & TECHNOLOGY

LOG BOOK

Name of the Staff Member: R.Prasad Rao Course: B.Tech II Subject: RVSP

Day & Date	No. of Periods	Cumulative No. of Periods	Topic (s) Covered in the Class
1/10/21	1	1	Introduction, Review of probability theory
4/10/21	1	2	Defnition of Random variable
5/10/21	1	3	conditions for a functions to be a R.V
6/10/21	1	4	Discrete, continuous and mixed Random variables
7/10/21	1	5	distribution and density functions
7/10/21	1	6	properties
8/10/21	1	7	Binominal, Poisson, uniform
9/10/21	1	8	Gaussian, exponential
11/10/21	1	9	Rayligh.
12/10/21	1	10	Conditional distribution
13/10/21	1	11	conditional density
13/10/21	1	12	Properties
14/10/21	1	13	expected value of Random variable
15/10/21	1	14	Function of a Random variable
16/10/21	1	15	moments about the orgin
18/10/21	1	16	central moments, variance and skew
19/10/21	1	17	chebychev's inequality
20/10/21	1	18	characterstic function.
21/10/21	1	19	moment generating function
21/10/21	1	20	transformation of Random variable
21/10/21	1	21	monotonic transformation for a continuous R.V
22/10/21	1	22	non monotonic transformation of continuous R.V
22/10/21	1	23	Rivison
23/10/21	1	24	Rivison.
25/10/21	1	25	vector random variables
26/10/21	1	26	Joint distribution function
27/10/21	1	27	Properties of Joint distribution, marginal distri
28/10/21	1	28	Conditional distribution and density
28/10/21	1	29	statistical impedance, sum of two Random. V
29/10/21	1	30	sum of several Random variables, General limit th
30/10/21	1	31	unequal distribution, equal distribution
1/11/21	1	32	Joint moment about the orgin, Joint General moment
1/11/21	1	33	Joint characterstic function, Joint gaussian R.V
2/11/21	1	34	Two Random variables case, N Random variables case
3/11/21	1	35	Properties transformation of multiple R.V, linear
4/11/21	1	36	the Random process concept.
5/11/21	1	37	Classification of process
5/11/21	1	38	deterministic and non deterministic Process
6/11/21	1	39	distribution and density function
8/11/21	1	40	Concept of stationary and statistical impedance

AVANTHI INSTITUTE OF ENGINEERING & TECHNOLOGY

LOG BOOK

Name of the Staff Member.....Course.....Subject .....

Day & Date	No. of Periods	Cumulative No. of Periods	Topic (s) Covered in the Class
9/11/21	1	41	Stationary process, second ordered W.S.S
10/11/21	1	42	Nth-order and strict - sense stationary
11/11/21	1	43	Time averages and ergodicity, auto correlation
12/11/21	1	44	Properties, Covariance functions.
13/11/21	1	45	Gaussian Random process
13/11/21	1	46	Poisson Random Process
15/11/21	1	47	the power density spectrum; properties
16/11/21	1	48	Relationship b/w power density spectrum and Auto correl.
17/11/21	1	49	the cross power density spectrum
18/11/21	1	50	Properties
19/11/21	1	51	Relation b/w cross power density spetrum and cross
20/11/21	1	52	Random signal Response of linear systems
22/11/21	1	53	System Response convolution
23/11/21	1	54	mean and mean squared value of system response
24/11/21	1	55	auto correlation function of Response
25/11/21	1	56	Cross correlation functions of input and output
26/11/21	1	57	spectral characteristics of system Response
27/11/21	1	58	Power density spectrum of Response
28/11/21	1	59	cross-power density spectrum i/p and o/p
29/11/21	1	60	Band pass
30/11/21	1	61	Band limited
1/12/21	1	62	Narrow band process
2/12/21	1	63	Revision
3/12/21	1	64	Rivision
14/12/21	1	65	properties
20/12/21	1	66	Revision
24/12/21	1	67	Revision
27/12/21	1	68	Revison
3/01/22	1	69	Revision
7/01/22	1	70	Revision.

AVANTHI INSTITUTE OF ENGINEERING & TECHNOLOGY

Tamaram, Makavarapalem, Narsipatnam Revenue Division, Visakhapatnam Dist-531113.

Addl. Code No.

Total Marks 29/30

MAIN ANSWER SHEET

MID EXAMINATION - I / II / III / IV Semester: I / II / III / IV / V

COURSES : B.Tech / MBA / M.Tech.

	Section A			Section B	
Q.No.	1	2	3	4	5
Marks					

Name: K. Bhargavi Subject: RVSP Date: 8-9-2022

Year & Branch: 2nd, ECE No. of Additionals: Roll No. 2 0 8 1 1 A 0 4 2 8

Signature of the Invigilator:

1. (a) Conditional distribution function :-

Let the A and B are the events, it provides $P(B) \neq 0$, the conditional probability density functions of event A, given by event B. $f(x/B)$ be the function, then the probability $f\left(\frac{x}{B}\right) = \frac{P(X \leq x)}{B} = \frac{P(X \leq x \cap B)}{P(B)}$.

If X be the discrete random variable then $F\left(\frac{x}{B}\right) = \sum_{x=0}^{\infty} P\left(\frac{x_i}{B}\right) \mu(x - x_i)$

If is a continuous random variable

$F(x/B) = \int_{-\infty}^{\infty} f_X\left(\frac{x}{B}\right) dx$.

5

Properties:

* $f\left(\frac{x}{B}\right)$ is a non-decreasing function of x.
* $f\left(\frac{-\infty}{B}\right) = 0$, It does not include real numbers less than $-\infty$.
* $f\left(\frac{\infty}{B}\right) = 1$. It is includes all real numbers.
* $0 \leq x \leq 1$, th it includes the real numbers.

$$* \; F\left(\frac{x}{B}\right) = \int_{-\infty}^{\infty} f_X\left(\frac{x}{B}\right) dx.$$

$$\frac{P(x_1 \le X \le x_2)}{P(B} = \int_{x_1}^{x_2} f_X\left(\frac{x}{B}\right) dx.$$

$$= F_X\left(\frac{x_2}{B}\right) - F_X\left(\frac{x_1}{B}\right)$$

1 (b) Gaussian random variable 'X' has mean $a_X = 2$, and variance $= 2$.

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x-m_x)^2}{2\sigma^2}}$$

To get the relation between $f_X(x)$ and $F(x)$

$$F_X(x) = F\left[\frac{x-m_2}{2}\right]$$

$$F_X[1.0] = F\left[\frac{1-2}{2}\right] = F\left[\frac{-3}{2}\right]$$

$$= F[-1.5]$$

$$Q = 1 - F[1.5]$$

$$Q \; Q(1.5)$$

$$Q(x) = \frac{1}{0.661x + 0.339\sqrt{x^2+5.51}} \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

$$= \frac{1}{0.661(1.5) + 0.339\sqrt{(1.5)^2+5.51}} \cdot \frac{e^{-(1.5)^2/2}}{\sqrt{2\pi}}$$

$$= \frac{1}{0.9915 + 0.339\sqrt{2.25+5.51}} \frac{e^{-(1.5)^2/2}}{\sqrt{2\pi}}$$

$$= \frac{1}{0.9915 + 0.339\sqrt{7.76}} \frac{e^{-(1.5)^2/2}}{\sqrt{2\pi}}$$

$$= \frac{1}{0.9915 + 0.9443} \cdot \frac{e^{-(1.125)}}{\sqrt{2\pi}}$$

$$= \frac{1}{3.8412} \Rightarrow P(X \le 1.0) = 1 - P(X \le 1.0)$$

$$= 1 - \frac{1}{3.812}$$

$$F_X(x) = F\left[\frac{x-m_x}{2}\right]$$

$$P(X > 1.5) = F\left[\frac{1-2}{2}\right] = F\left[\frac{-1}{2}\right] = F[-0.5]$$

5

$$Q = 1 - F(0.5)$$

$$Q(0.5)$$

$$Q(x) = \frac{1}{0.661x + 0.339\sqrt{x^2+5.59}} \frac{e^{-(x^2/2)}}{\sqrt{2\pi}}$$

$$= \frac{1}{0.661(0.5) + 0.339\sqrt{(0.5)^2+5.59}} \frac{e^{-(0.5)^2/2}}{\sqrt{2\pi}}$$

$$= \frac{1}{0.3305 + 0.339\sqrt{0.25+5.59}} \frac{e^{-0.25/2}}{\sqrt{2\pi}}$$

$$= \frac{1}{0.3305 + 0.339\sqrt{5.84}} \cdot \frac{e^{-0.125}}{2.506}$$

$$= \frac{1}{0.3305 + 0.81923} \cdot \frac{e^{-0.125}}{2.506}$$

$$= 0.3085$$

$$P(X > 1.0) = 1 - F_X(1.0)$$

$$= 1 - Q(0.5)$$

$$= 0.6915$$

---

2 (b) Uniform density function

$$f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise.} \end{cases}$$

Mean:-

$$\text{Mean} = E[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx.$$

$$= \int_{-\infty}^{a} x f_X(x)\, dx + \int_{a}^{b} x f_X(x)\, dx + \int_{b}^{\infty} x f_X(x)\, dx$$

$$= 0 + \int_{a}^{b} x f_X(x)\, dx + 0$$

$$= \int_{a}^{b} x f_X(x)\, dx.$$

$$= \int_{a}^{b} x \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a}\int_a^b x\,dx$$

$$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{1}{b-a}\left[\frac{b^2}{2} - \frac{a^2}{2}\right]$$

$$= \frac{1}{b-a}\left[\frac{b^2-a^2}{2}\right]$$

$$= \frac{1}{\cancel{b-a}}\left[\frac{\cancel{(b-a)}(b+a)}{2}\right]$$

$$= \frac{b+a}{2}.$$

ii) $\text{variance}[x] = E[x^2] - \{E[x]\}^2$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx.$$

$$= \int_{-\infty}^{a} x^2 f_X(x)\,dx + \int_a^b x^2 f_X(x)\,dx + \int_b^{\infty} x^2 f_X(x)\,dx$$

$$0 + \int_a^b x^2 \frac{1}{b-a}\,dx + 0$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$$= \frac{1}{b-a}\left[\frac{b^3}{3} - \frac{a^3}{3}\right]$$

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$$= \frac{1}{b-a}\left[\frac{(b^3-a^3)}{3}\right]$$

$$= \frac{1}{\cancel{b-a}}\left[\frac{\cancel{(b-a)}(b^2+ab+a^2)}{3}\right]$$

$$= \frac{b^2+ab+a^2}{3}.$$

$$\text{variance}[x] = E[x^2] - \{E[x]\}^2$$

$$= \frac{b^2+ab+a^2}{3} - \left(\frac{b+a}{2}\right)^2$$

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$$= \frac{2(b^2+ab+b^2) - 3(b+a)}{6}$$

$$= \frac{2b^2+2ab+2b^2-3b-3a}{6}$$

$$= \frac{b^2+ab+a^2}{3} - \frac{b^2+a^2+2ab}{4}$$

$$= \frac{4(b^2+ab+a^2) - 3(b^2+a^2+2ab)}{12}$$

$$= \frac{4b^2+4ab+4a^2-3b^2-3a^2-6ab}{12}$$

$$= \frac{b^2+a^2-2ab}{12}$$

$$= \frac{a^2+b^2+4ab}{6} \qquad \frac{(b-a)^2}{12} \quad (\text{or}) \quad \frac{(a-b)^2}{12}$$

$$= \frac{(a+b)^2}{6}$$

2 (b) $f_X(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$

Given $f_X x = \begin{cases} \frac{1}{b-a} & -\pi/2 \le x \le \pi/2 \\ 0 & \text{otherwise} \end{cases}$

New random variable $= y = a\tan(x)$

$$\frac{y}{a} = \tan(x)$$

$$x = \tan^{-1}\left(\frac{y}{a}\right)$$

differentiate the above equation

$$\frac{dx}{dy} = \frac{1}{\left(1+\left(\frac{y}{a}\right)^2\right)} \times \frac{1}{a}$$

from the abe $\frac{1}{b-a}$ $\quad -\pi/2 \le x \le \pi/2$

$$= \frac{1}{\frac{\pi}{2} - (-\pi/2)}$$

$$= \frac{1}{\frac{\pi}{2}+\frac{\pi}{2}}$$

$$= \frac{1}{\frac{2\pi}{2}}$$

$$= \frac{1}{\pi}$$

$$\frac{dx}{dy} = \frac{1}{1+\frac{y^2}{a^2}} \times \frac{1}{a}$$

$$= \frac{a^2}{a^2+y^2} \times \frac{1}{a}$$

$$= \frac{a}{a^2+y^2}$$

The probability density function of y =

$$f_Y(y) = f_X(x)\frac{dx}{dy}$$

$$= \frac{1}{\pi}\left(\frac{a}{a^2+y^2}\right)$$

3. (b) Given marginal or density functions

$$f_X(x) = 4u(x)e^{-4x}$$

$$f_Y(y) = 5\mu(y)e^{-5y}$$

Given density function $w = X+Y$

The x and Y are independent functions

$$f_W(w) = f_{X+Y}(w) = f_X(x) * f_Y(y)$$

(or)

$$f_Y(y) \text{ or } f_X(x)$$

$$\int_{-\infty}^{\infty} f_X(x) f_Y(w-x)\,dx \quad \text{or} \quad \int_{-\infty}^{\infty} f_Y(y) f_X(w-y)\,dy.$$

consider

$$\int_{-\infty}^{\infty} f_X(x) f_Y(w-x)\,dx.$$

$$= \int_0^{w} 4u(x)e^{-4x}\, 5u(w-x)e^{-5(w-x)}\,dx.$$

$$= \int_0^{w} 4e^{-4x}\, 5e^{-5(w-x)}\,dx$$

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$$= 20\int_0^{\omega} e^{-4x} e^{-5\omega} e^{5x}\, dx.$$

$$= 20e^{-5\omega}\int_0^{\omega} e^{x}\, dx$$

$$= 20e^{-5\omega}\left[e^{x}\right]_0^{\omega}$$

$$= 20e^{-5\omega}\left[e^{\omega} - e^{0}\right]$$

$$= 20\left[e^{-4\omega} - e^{-5\omega}\right]$$

3 (a)

$$f_{XY}(x,y) = \frac{1}{12}\mu(x)\mu(y)e^{-x/3}e^{-y/4}$$

If X and Y are independent then.

$$= \frac{1}{12}\int_{-\infty}^{0} \mu(x)\,\mu(y)\, e^{-x/3} e^{-y/4}\, dy$$

$$= \frac{1}{12}\int_{-\infty}^{0} (1)(1)\, e^{-x/3} e^{-y/4}\, dy.$$

$$= \frac{e^{-x/3}}{12}\int_{-\infty}^{0} e^{-y/4}\, dy$$

$$= \frac{e^{-x/3}}{12}\left[\frac{4e^{-y/4}}{4}\right]_{-\infty}^{0}$$

$$= \frac{e^{-x/3}}{12}\left[\frac{e^{0}}{4} - \frac{e^{+\infty}}{4}\right] \qquad \Rightarrow \frac{e^{-x/3}}{12}\left[4e^{-(y)/4}\right]_{-\infty}^{0}$$

$$= \frac{e^{-x/3}}{12}\left[\frac{1}{4} - \frac{0}{4}\right] = \frac{e^{-x/3}}{3}$$